Upsetting the Fine Structure Constant without 12672 Diagrams

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Outline

- Fine Structure Constant and the Anomalous Magnetic Moment of the Electron.
- Recent Calculations and Measurements of a_e
- Hadronic Contributions
- New Hadronic Bound State
- Contributions of New Hadrons to a_e , α

Fine Structure Constant

•The most recent **measurement** (Hanneke, Fogwell, Gabrielse, 2008):

$$\alpha^{-1} = 137.035999084(51)$$

•Fine structure constant related to the electron's magnetic moment anomaly

•Most accurate measurement of α from electron's anomalous magnetic moment.

Measurments / Calculations of a_e

•Recent **measurement** of a_e (Hanneke, Fogwell, Gabrielse, 2008):

$$a_e = (1159652180.73 \pm 0.28) \cdot 10^{-12}$$

•Recent **calculation** of a_e (eight order) (Aoyama, Hayakawa, Kinoshita, Nio, 2008):

$$a_e(\mathrm{Rb}) = (1159652182.79 \pm 7.71) \cdot 10^{-12}$$

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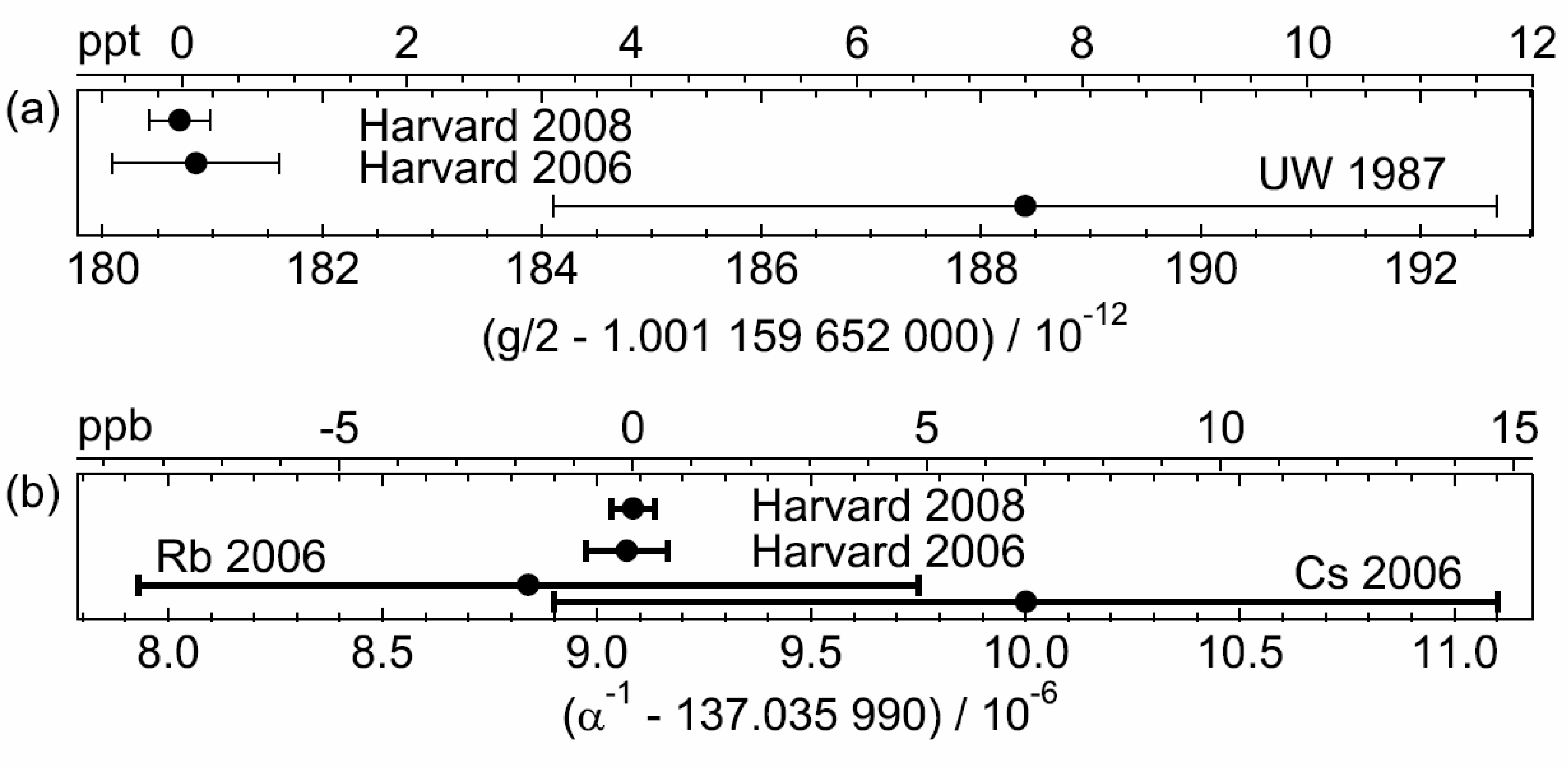

Hanneke, Fogwell, Gabrielese, 2008

a_e

- Magnetic moment anomaly usually calculated as:

$$a_e = a_e(QED) + a_e(hadron) + a_e(weak)$$

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Hadronic Contributions

- Dispersion Integral:

$$a_\mu^{had,LO} = \frac{\alpha^2}{3\pi^2} \int_{s_{min}}^{\infty} \mathrm{d}s \frac{K(x(s))}{s} R(s),$$

$$K(x) = x^2\left(1 - \frac{x^2}{2}\right) + (1+x)^2\left(1 + \frac{1}{x^2}\right)\left(\ln(1+x) - x + \frac{x^2}{2}\right) + \frac{1+x}{1-x} x^2 \ln x,$$

$$x(s) = \frac{1-\beta_e}{1+\beta_e}$$

$$\beta_e = \sqrt{1 - 4m_e/s}$$

$$R(s) = \frac{\sigma(e^+e^- \rightarrow hadrons)}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)}$$

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KU

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Will come back to this.

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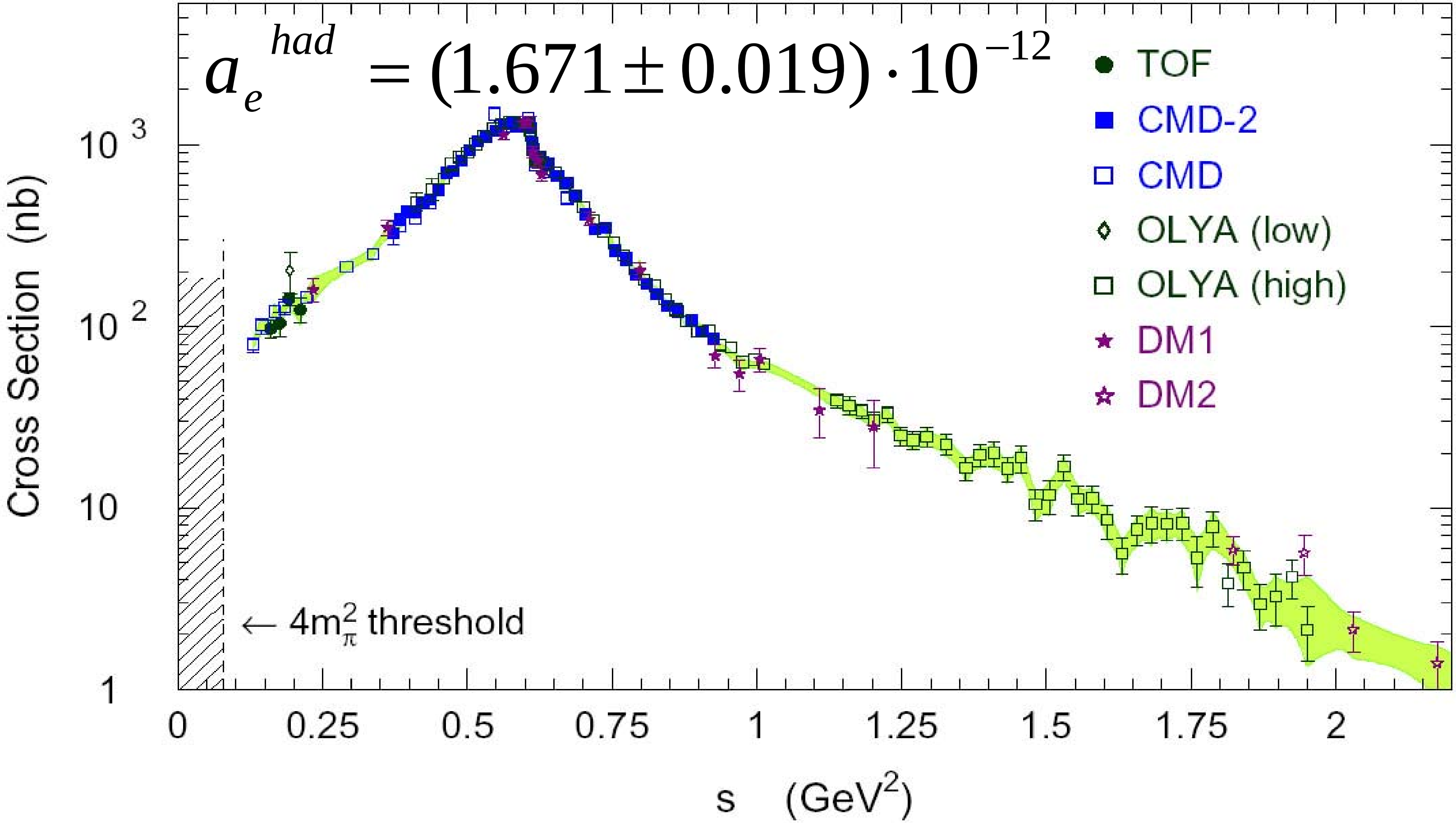
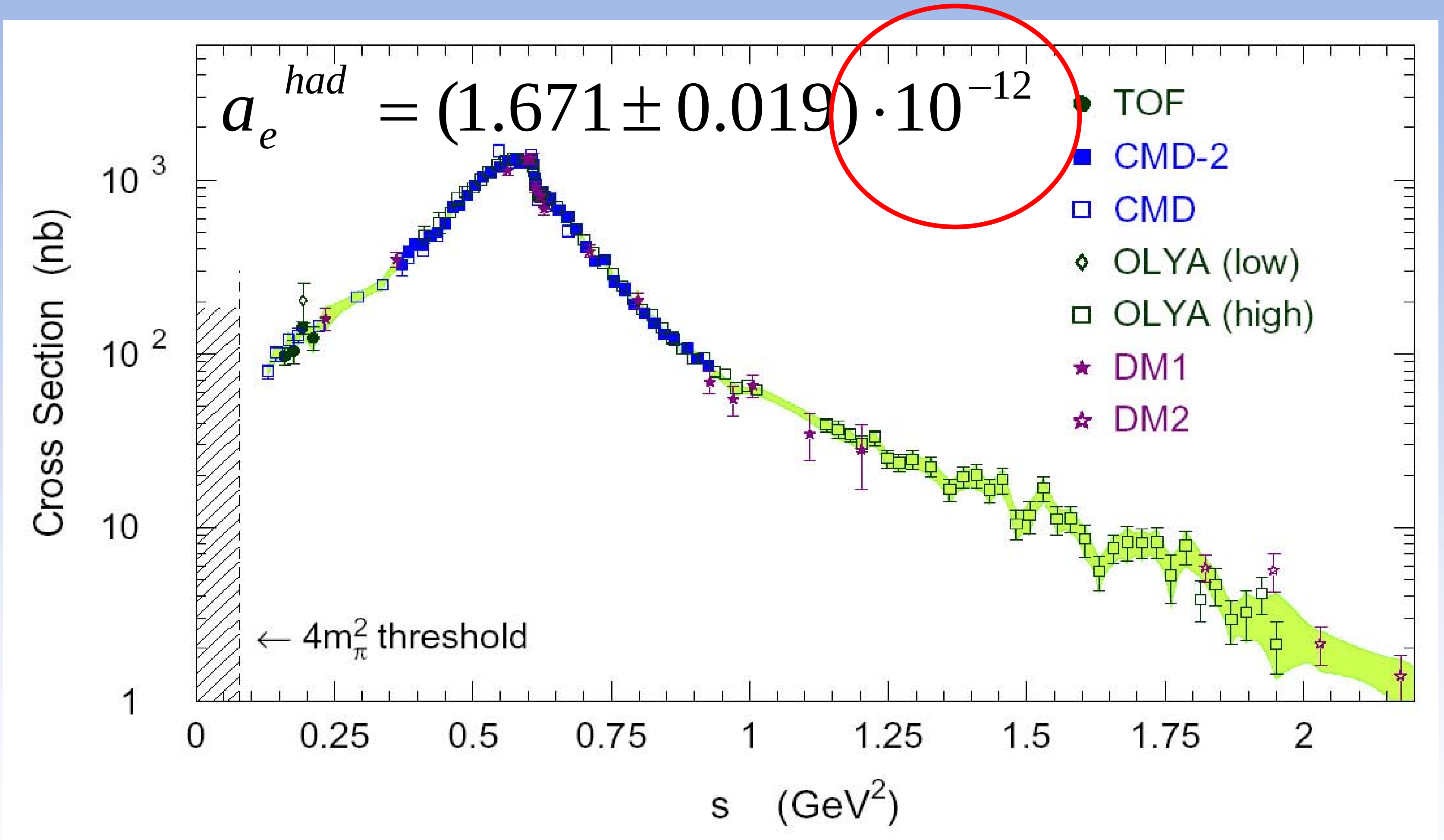
Hadronic Contributions

$$a_e^{had} = (1.671 \pm 0.019) \cdot 10^{-12}$$



Davier et al, 2002

Hadronic Contributions



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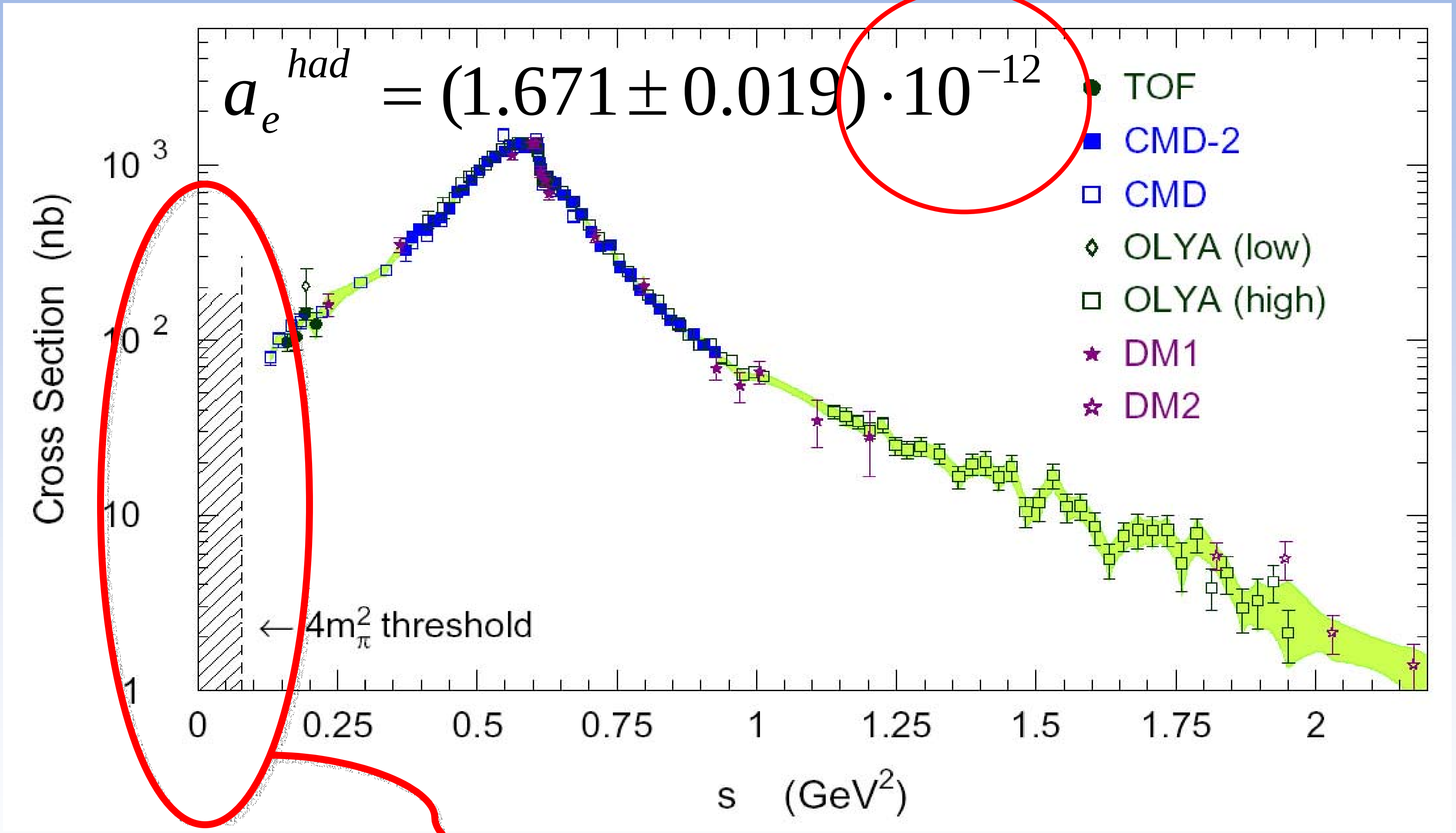
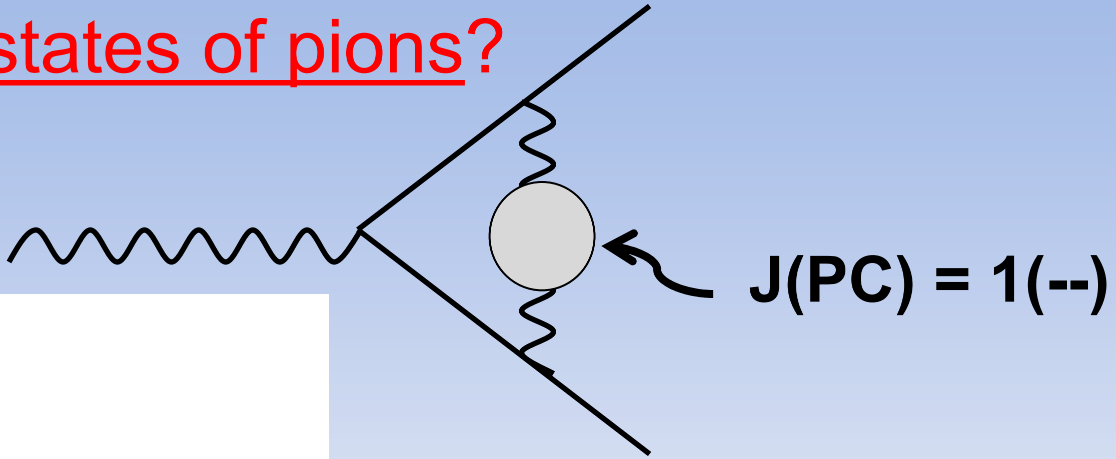


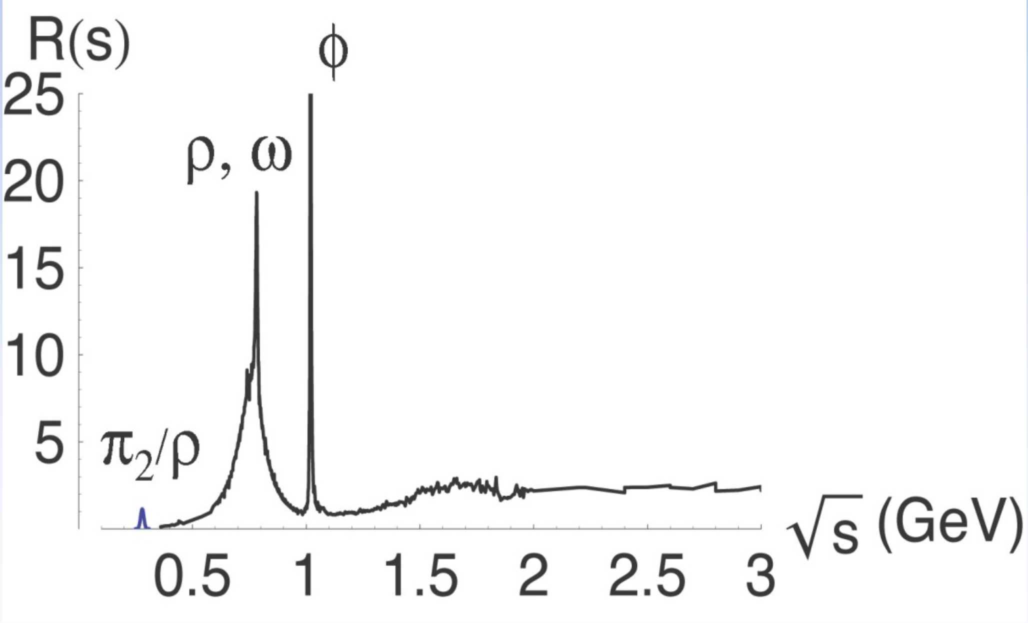
Davier et al, 2002

No data for low energies!!!!!

Hadronic Contributions

- What about bound states of pions?





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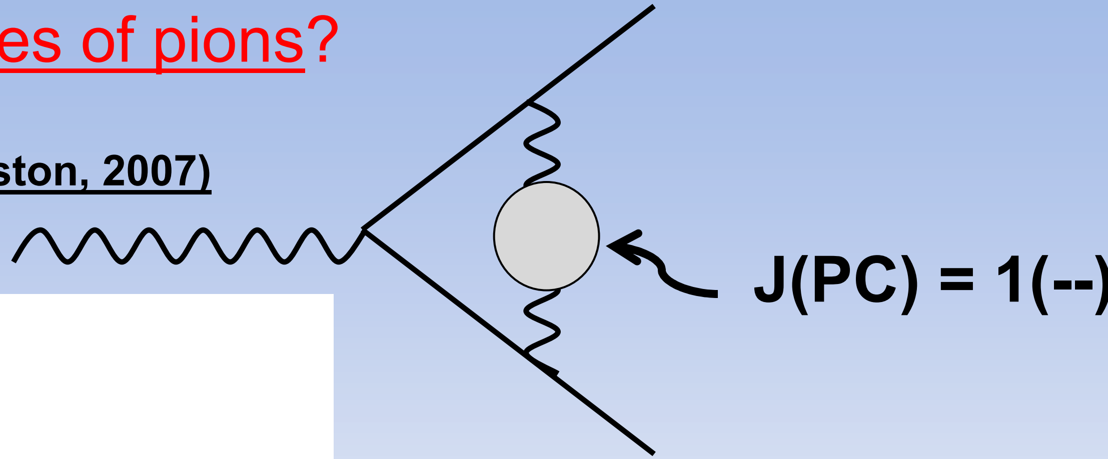
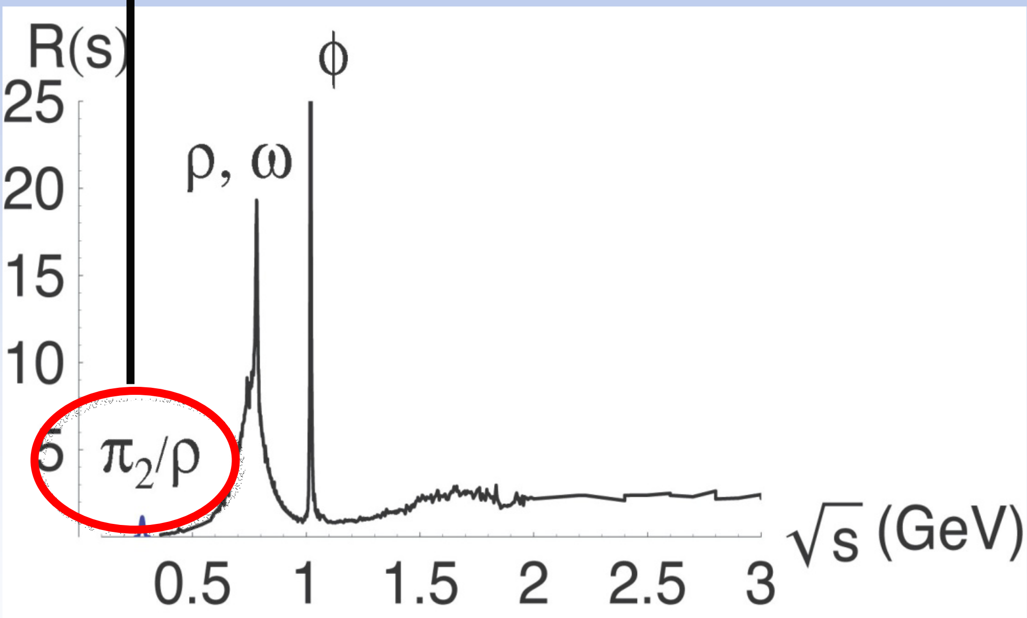
Pi-rhonium (Schiel, Ralston, 2007)





Explains the 3σ discrepancy in muon's anomalous magnetic moment.

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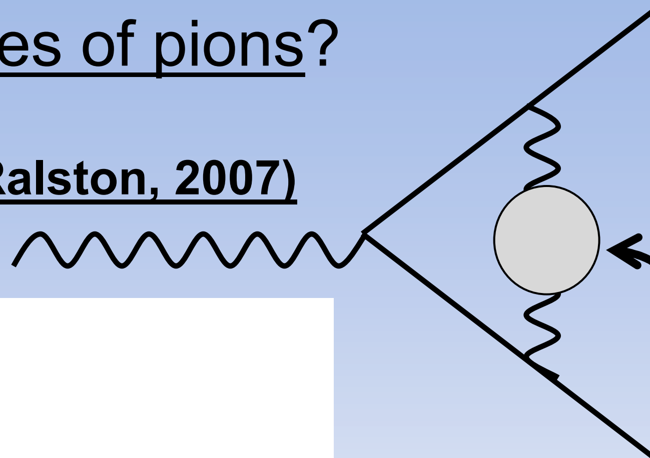
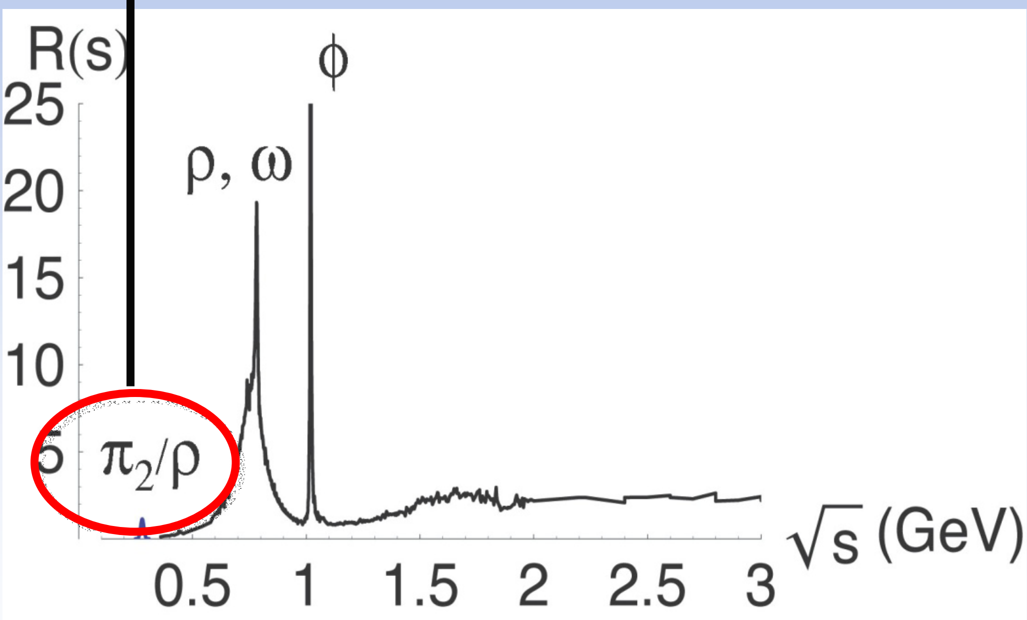


J(PC) = 1(--)



Pionic atoms (π+π-) have already been observed (DIRAC experiment)

Explains the 3σ discrepancy in muon’s anomalous magnetic moment.

Pi-rhonium Contribution

Use the Breit-Wigner formula to obtain:

$$a_e^{\pi_{2/\rho}} = \frac{3}{\pi} \frac{K(x(m^2_{\pi_{2/\rho}}))}{m_{\pi_{2/\rho}}} \Gamma(\pi_{2/\rho} \to e^+ e^-)$$

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28eV if the state amounts for the full discrepancy of the muon's anomalous magnetic moment. (Schiel, Ralston, 2007)

Unknown. No data

Mixes with ρ To about 1%

Pi-rhonium Contribution

- So… if $\Gamma(\pi_{2/\rho} \rightarrow e^{+}e^{-}) = 28eV$ then…

$$a_e^{\pi 2/\rho} = (4.74049 \pm 0.3) \cdot 10^{-13}$$

Significant at next order!

Add this contribution to the fine structure constant calculation to get:

$$\alpha^{-1} = 137.0359991399(51)(4)$$

Conclusions

- Excited state of a pionic atom could contribute to the next order of accuracy of the fine structure constant.
- Need data in the lower energy part of R(s).
- Nonperturbative methods?
- Next order calculation /experiment should consider this contribution