Phase transition in the fine structure constant

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with Anchordoqui, Barger and Goldberg (0711.4055)

Mass-varying neutrinos

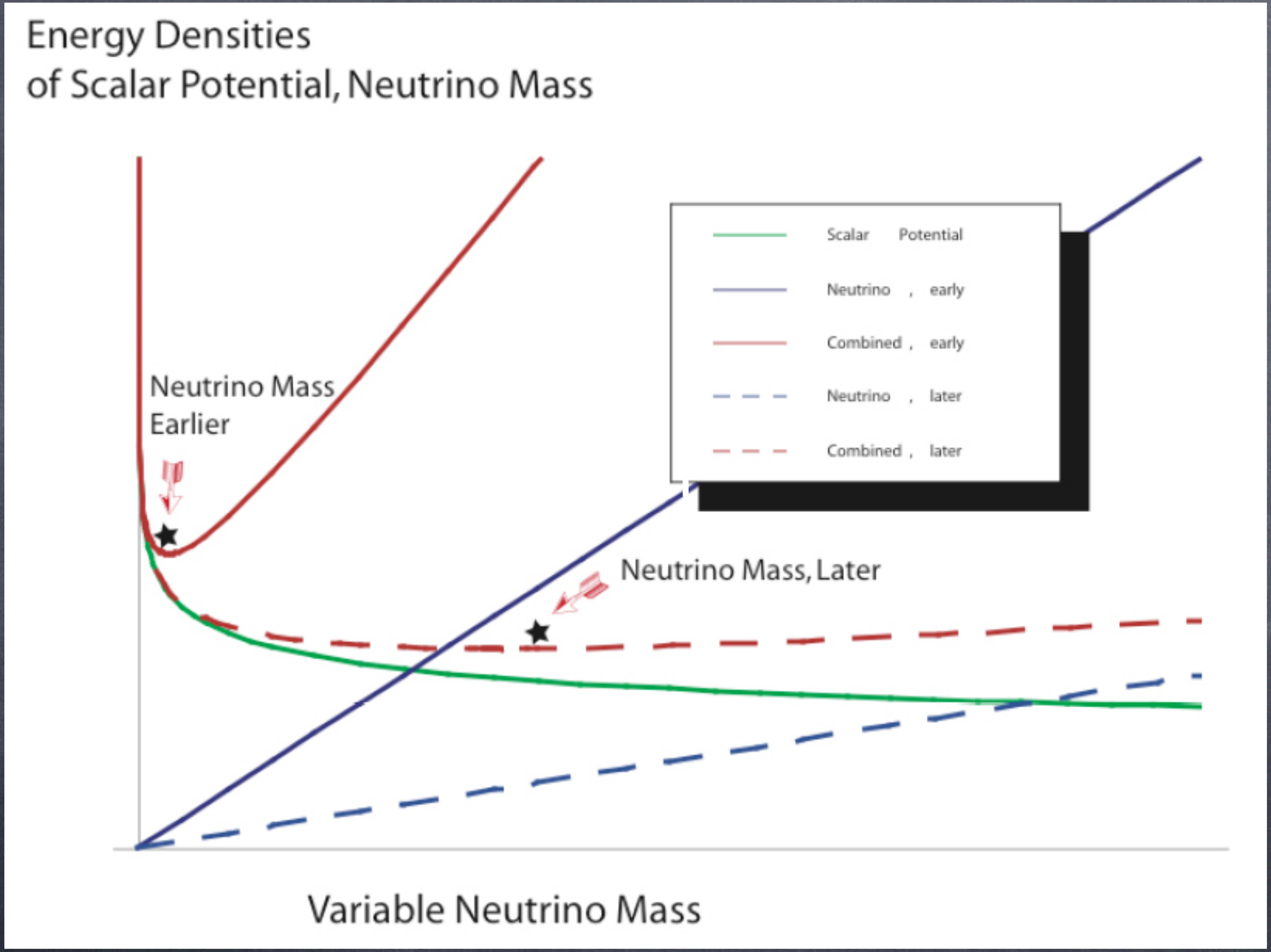
$$\rho_{\rm DE} \sim 3M_{\rm Pl}^2 H_0^2 \sim (2.4 \times 10^{-3}\ {\rm eV})^4$$

$$\delta m^2 \sim (10 \times 10^{-3} {\rm eV})^2$$

Coupling neutrinos to a light scalar may explain

$\Omega_{\rm DE} \sim \Omega_M$ Fardon, Nelson, Weiner

Minimize $V_{\rm eff}(m_\nu) = m_\nu n_\nu + V(m_\nu)$ wrt m_ν



Nelson

Status (from low to high redshifts)

- Measurements of transition frequencies in atomic clocks give the limit $|\Delta\alpha/\alpha| < 5 \times 10^{-15}$

- Abundance ratio of Sm-149 to Sm-147 at the Oklo natural reactor shows no variation in the last 1.7 Gyr: $|\Delta\alpha/\alpha| < 10^{-7}$

- Meteoritic data (z < 0.5) constrain the beta-decay rate of Re-187 back to the time of solar system formation (4.6 Gyr): $\Delta\alpha/\alpha = (8 \pm 8) \times 10^{-7}$

- Comparison of transition lines in QSO spectra (0.5 < z < 4) indicate $\Delta\alpha/\alpha = (-0.57 \pm 0.10) \times 10^{-5}$

- Measurements of the CMB (z = 1100) accurately determine the temperature at decoupling which depends on the binding energy of hydrogen. Current constraint is $|\Delta\alpha/\alpha| < 0.02$

- Primordial abundances from BBN (z = 10 billion) depend critically on the neutron-proton mass difference which depends on alpha. Current limit: $|\Delta\alpha/\alpha| < 0.02$

Phase transition in alpha

$$\begin{aligned} V_{\text{eff}}^{\text{NR}} &= m_\nu(\mathcal{A})\ n_\nu + V[M(\mathcal{A})] \\ &= \frac{m_D^2}{M(\mathcal{A})}\ n_\nu\ + V[M(\mathcal{A})] \end{aligned}$$

$$\begin{aligned} V_{\text{eff}}^{\text{REL}} &= m_\nu(\mathcal{A})^2\ \frac{n_\nu}{\langle E_\nu \rangle} + V[M(\mathcal{A})] \\ &= \frac{m_D^4}{\langle E_\nu \rangle M(\mathcal{A})^2}\ n_\nu + V[M(\mathcal{A})] \end{aligned}$$

Stationary points given by

$$\frac{dV_{\rm eff}^{\rm NR}}{d\mathcal{A}} = \left(-\frac{m_D^2\, n_\nu}{M^2} + V'(M) \right) \frac{dM}{d\mathcal{A}} \;=\; 0$$

$$\frac{dV_{\rm eff}^{\rm REL}}{d\mathcal{A}} = \left(-\frac{2 m_D^4\, n_\nu}{\langle E_\nu \rangle\, M^3} + V'(M) \right) \frac{dM}{d\mathcal{A}} \;=\; 0$$

$$\mathrm{V}'(M) \equiv \partial V(M)/\partial M$$

Assumption I: M has a unique stationary point

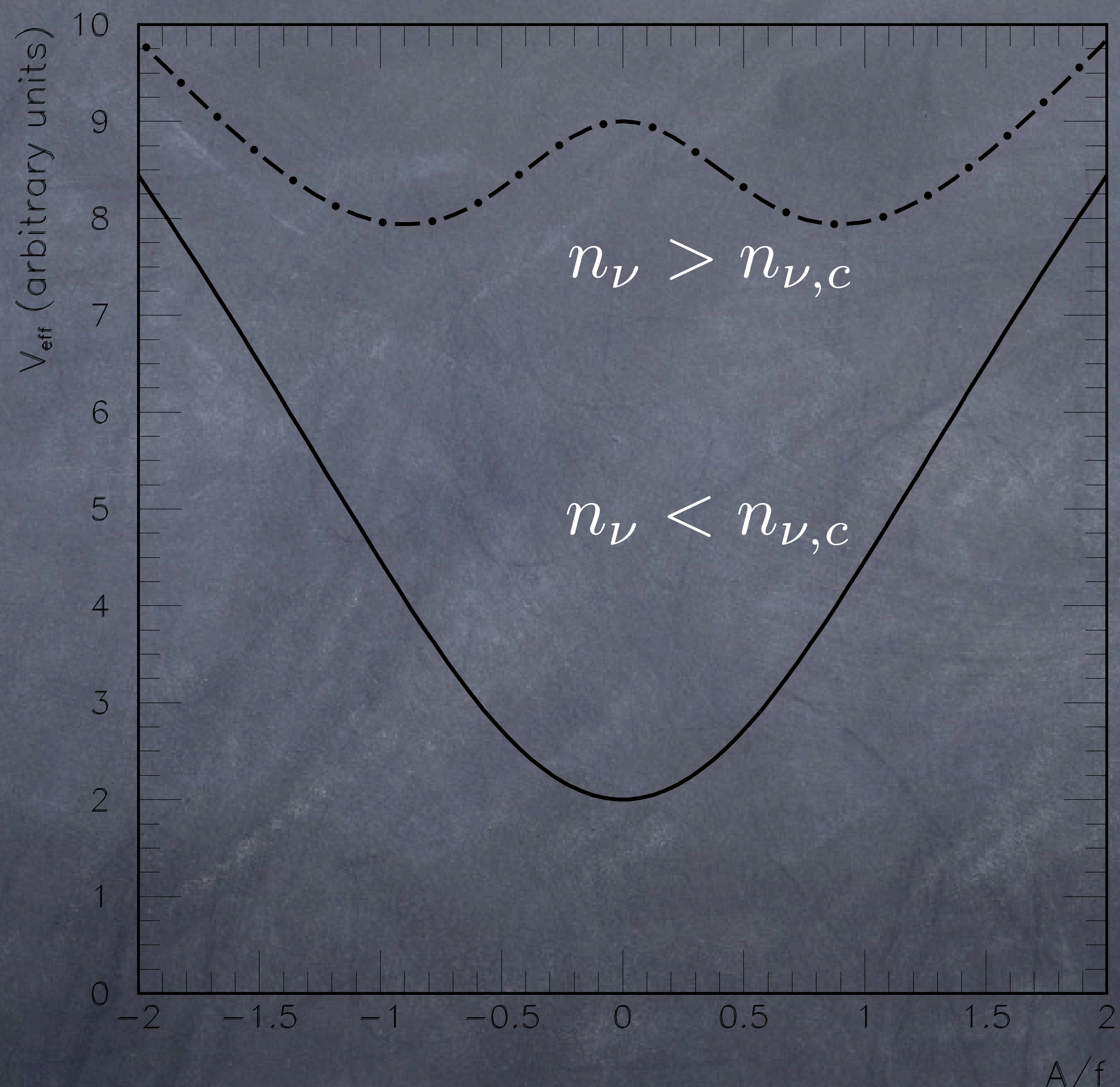
$M(\mathcal{A}) \simeq M_o[1 + \mathcal{A}^2/f^2]$ in the vicinity of the min

Additional stationary points will exist if

$$\left(\frac{M}{M_o}\right)^j \frac{V'(M)}{V'(M_o)} = \frac{n_\nu}{n^i_{\nu,\mathrm{c}}} \qquad j = 2, 3 \text{ if } i = \mathrm{NR}, \mathrm{REL}$$

$$n^{\mathrm{NR}}_{\nu,\mathrm{c}} \equiv \frac{M_o^2}{m_D^2}\, V'(M_o)\,, \qquad n^{\mathrm{REL}}_{\nu,\mathrm{c}} \equiv \frac{M_o^3}{2\, m_D^4}\, V'(M_o)$$

Assumption II: $M^2 V'(M)$ is an increasing fn of M



Example

$$V[M(\mathcal{A})] = \Lambda^4 \ \ln(|M(\mathcal{A})/M_o|)$$

$$\left(\frac{M}{M_o}\right)^k = \frac{n_\nu}{n^i_{\nu,\mathrm{c}}}$$

$$k = 1, 2 \text{ for } i = \mathrm{NR}, \mathrm{REL}$$

$$n^{\mathrm{NR}}_{\nu,\mathrm{c}} = \frac{\Lambda^4}{m_{\nu,0}}\,, \qquad n^{\mathrm{REL}}_{\nu,\mathrm{c}} = \frac{\langle E_\nu \rangle \ \Lambda^4}{2\, m^2_{\nu,0}} \simeq \frac{T_\nu \ \Lambda^4}{m^2_{\nu,0}}$$

$$m_{\nu,0} \equiv m^2_D / M_o$$

For nonrelativistic neutrinos with subcritical neutrino density, the only stationary point is $A = 0$ with $M = M_o$

$$\implies m_\nu = m_{\nu,0}$$

No stability issues because neutrino mass is independent of neutrino density

For nonrelativistic neutrinos with supercritical neutrino density,

$$m_\nu = \Lambda^4 / n_\nu$$

Window of instability

Acceleron mediates an attractive force between neutrinos which can form nuggets that behave like CDM

Afshordi, Zaldarriaga, Kohri

$$1.8\left(\frac{\Lambda}{m_{\nu,0}}\right)^{1/3} \lesssim \frac{T_\nu}{\Lambda} \lesssim 1.1$$

$$2.9\left(\frac{\Lambda_{-3}^4}{m_{\nu,0}/0.05\ \mathrm{eV}}\right)^{1/3} \lesssim 1+z \lesssim 6.5\ \Lambda_{-3}$$

$$\Lambda_{-3} \equiv \Lambda/(10^{-3}\ \mathrm{eV})$$

The instability is avoidable ...

... if growth-slowing effects (dragging) provided by CDM dominate over the acceleron-neutrino coupling

Bjaelde et al.

$$\beta \equiv \left|\frac{d\ln m_\nu}{d\mathcal{A}}\right| = \left|\frac{d\ln M}{d\mathcal{A}}\right| < \sqrt{\frac{\Omega_{\rm CDM} - \Omega_\nu}{2\,\Omega_\nu}}\,\frac{1}{M_{\rm Pl}} \simeq \frac{10}{M_{\rm Pl}}$$

If $M = M_o\, e^{\mathcal{A}^2/f^2} \implies \beta = \frac{2|\mathcal{A}|}{f^2} < \frac{10}{M_{\rm Pl}}$

$$T_\nu < 1.1\Lambda \implies \frac{|\mathcal{A}|}{f} < \sqrt{\ln\frac{12\ (m_{\nu,0}/0.05\ {\rm eV})}{\Lambda_{-3}}} < 1.7$$

for $\Lambda_{-3} \simeq 0.6(m_{\nu,0}/0.05\ {\rm eV})^{1/4}$

$$\implies f/M_{\rm Pl} > 0.34$$

Discontinuity in alpha

$$\mathcal{L}_{\rm em} = -\frac{1}{4}\, Z_F(\mathcal{A}/M_{\rm Pl})\, F_{\mu\nu}F^{\mu\nu}$$

$$= -\frac{1}{4}\,(1 + \kappa\, \mathcal{A}/M_{\rm Pl}\ + \ldots)\, F_{\mu\nu}F^{\mu\nu}$$

$$\kappa \equiv \partial_{\mathcal{A}} Z_F|_0$$

$$\left|\frac{\Delta\alpha}{\alpha}\right| = \kappa\frac{\mathcal{A}}{M_{\rm Pl}} = \kappa\ \frac{\mathcal{A}}{f}\cdot\frac{f}{M_{\rm Pl}}$$

Accommodating null low redshift data

Requiring that the acceleron not vary from its ground state till z = 0.5, so that alpha does not vary, gives

$$\Lambda_{-3} \simeq 0.61 (m_{\nu,0}/0.05 \text{ eV})^{1/4}$$

$$\frac{\rho_{\mathcal{A}}}{\rho_{\mathrm{DE}}} \sim 4 \times 10^{-3} \; \frac{m_{\nu,0}}{0.05 \text{ eV}}$$

The energy density of the acceleron does not saturate the present dark energy

Neutrinos are nonrelativistic with supercritical density for $0.5 < z < 4$

Reproducing the signal in quasar spectra

$$\frac{M(\mathcal{A})}{M_o} = \left(\frac{1+z}{1+z_c}\right)^3 \qquad \Longrightarrow \quad \frac{|\mathcal{A}|}{f} = \sqrt{3\ \ln\left(\frac{1+z}{1+z_c}\right)}$$

For $z = 2\,,\, z_c = 0.5$ $\qquad |\mathcal{A}|/f \simeq 1.4$

With $f/M_{\rm Pl} \geq 0.34$

$$\left|\frac{\Delta\alpha}{\alpha}\right| \gtrsim 0.5\ \kappa$$

Need $\kappa \sim 10^{-5}$ to explain the data

Consistent with CMB and BBN data?

$n_\nu > n_{\nu,\mathrm{c}}^{\mathrm{REL}}$ as soon as neutrinos become relativistic

$$\frac{M}{M_o} = \sqrt{\frac{3\zeta(3)}{2\pi^2}\,\frac{T_\nu^2\; m_{\nu,0}^2}{\Lambda^4}} \qquad \Longrightarrow \quad |\mathcal{A}|/f \simeq \sqrt{\ln\left(10\, z\right)}$$

CMB

$$\left|\frac{\Delta\alpha}{\alpha}\right| \simeq \frac{3\,\kappa\, f}{M_{\mathrm{Pl}}}\,,$$

BBN

$$\left|\frac{\Delta\alpha}{\alpha}\right| \simeq \frac{5\,\kappa\, f}{M_{\mathrm{Pl}}}$$

$$\begin{array}{llll} \kappa & < & 0.02 & \text{recombination} \\ \kappa & < & 0.01 & \text{BBN} \end{array}$$