Not T-parity but C-parity

(in the Littlest Higgs model with T-Parity)

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Pheno'08 Symposium
(2008.04.28-30)

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- **Review of Little Higgs and T-parity**
- Higgs potential and C-parity
- Origin of C-parity
- UV completions and gauge anomalies
- Summary and outlook

Review of Little Higgs and T-parity

- There are a lot of models for EWSB on the market. We talk about the Little Higgs theories.
- Among various Little Higgs models, we concentrate only on the **Littlest Higgs model**.
- To review the littlest Higgs model (with or without T-parity) look over the previous two talks in the session.

Review of Little Higgs and T-parity

- A global SU(5) is broken to SO(5) at a scale f.
  - a tensor field Σ $f \gtrsim 1\ \mathrm{TeV}$

$$VEV \quad \Sigma_0 = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

  - 14 Nambu-Goldstone bosons (NGBs)
- relevant gauge groups (ignore SU(3)c)

$$[SU(2) \times U(1)]_1 \times [SU(2) \times U(1)]_2 \Rightarrow SU(2)_L \times U(1)_Y$$

Review of Little Higgs and T-parity

- 14 NGBs = 4 + (4 + 6) NGBs
  - 4 eaten by the broken gauge fields of SU(2)xU(1)
  - 4 doublet under SU(2)L $H(x)$
  - 6 triplet under SU(2)L $\Phi(x)$
- (4 + 6) NGBs ⇨ nonlinear sigma field

$$\Sigma(x) = \exp\{2i\Pi(x)/f\}\Sigma_0$$

$$\Pi = \begin{pmatrix} 0 & H^T/\sqrt{2} & \Phi \\ H^*/\sqrt{2} & 0 & H/\sqrt{2} \\ \Phi^\dagger & H^\dagger/\sqrt{2} & 0 \end{pmatrix}$$

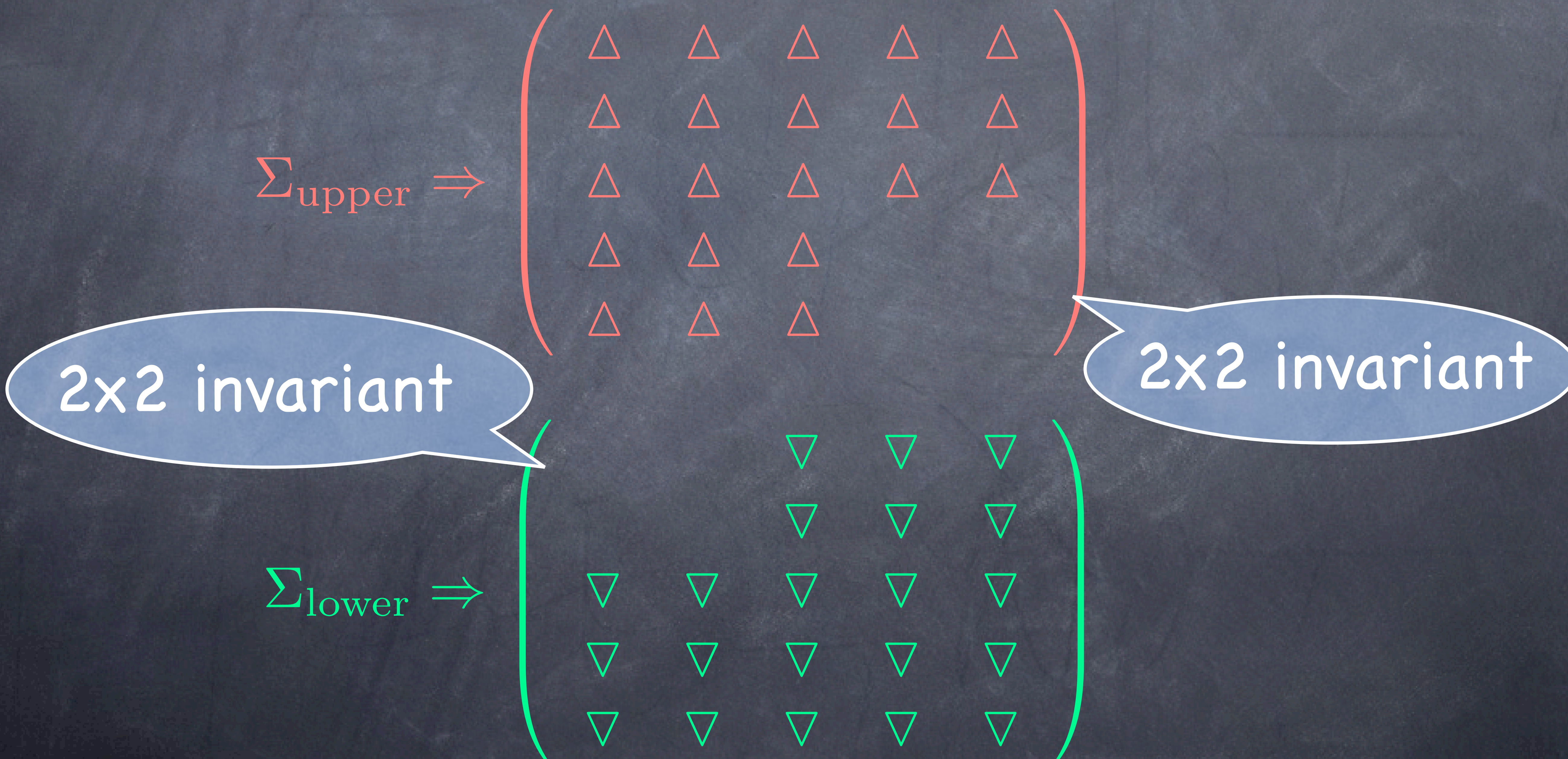
Review of Little Higgs and T-parity

- Collective symmetry breaking ?
  - global symmetry V ∈ SU(5), $\Sigma \rightarrow V \Sigma V^T$
  - subgroups : $SU(3)_{upper}$ and $SU(3)_{lower}$

$$V = \begin{pmatrix} \triangle & \triangle & \triangle & & \\ \triangle & \triangle & \triangle & & \\ \triangle & \triangle & \triangle\nabla & \nabla & \nabla \\ & & \nabla & \nabla & \nabla \\ & & \nabla & \nabla & \nabla \end{pmatrix}$$

Review of Little Higgs and T-parity

transformation $\Sigma \to V \Sigma V^T$



Review of Little Higgs and T-parity

- The two global symmetries are broken by gauge and Yukawa interactions. One particle breaks the $SU(3)_{upper}$ global symmetry and its partner particle with the same statistics does the $SU(3)_{lower}$ global symmetry through either gauge or Yukawa interactions.

- Only if both $SU(3)_{upper}$ and $SU(3)_{lower}$ are broken simultaneously the SM Higgs, H, acquires a mass so EWSB is triggered. This mechanism was dubbed "**collective symmetry breaking**."

Review of Little Higgs and T-parity

- To meet EWPT the VEV scale, f, must be lifted up to 10 TeV. Therefore, the fine-tuning of the SM Higgs mass appears once again.
- T-parity is introduced to overcome this undesirable fine-tuning problem.
- T-odd particles are created or annihilated in pairs and the lightest T-odd particle (LTOP), the heavy photon, is the candidate for Dark Matter.

T-parity in Higgs sector

$$T : \begin{cases} H & \rightarrow & H \\ \Phi & \rightarrow & -\Phi. \end{cases}$$

T-parity in gauge sector

$$T \; : \; W_1^a \leftrightarrow W_2^a, \qquad B_1 \leftrightarrow B_2$$

$$\Rightarrow \; g_1 = g_2 = \sqrt{2}g, \quad g_1' = g_2' = \sqrt{2}g'$$

$$\begin{aligned} W_H^a &= \frac{1}{\sqrt{2}}(W_1^a - W_2^a), \quad W_L^a = \frac{1}{\sqrt{2}}(W^a + W_2^a) \\ B_H &= \frac{1}{\sqrt{2}}(B_1 - B_2), \quad B_L = \frac{1}{\sqrt{2}}(B_1 + B_2) \end{aligned}$$

T-parity in fermion sector

$$\Psi_1 = \begin{pmatrix} \psi_1 \\ 0 \\ 0 \end{pmatrix}, \qquad \Psi_2 = \begin{pmatrix} 0 \\ 0 \\ \psi_2 \end{pmatrix}$$

ψ_1 doublet under $SU(2)_1$

ψ_2 doublet under $SU(2)_2$

$$T \; : \; \psi_1 \leftrightarrow -\psi_2 \quad \left(\Psi_1 \leftrightarrow -\Sigma_0 \Psi_2\right)$$

$$\begin{aligned} \psi_+ &= (\psi_1 - \psi_2)/\sqrt{2} \quad \text{T-even} \\ \psi_- &= (\psi_1 + \psi_2)/\sqrt{2} \quad \text{T-odd} \end{aligned}$$

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Higgs potential and C-parity

Higgs potential from gauge interaction

$$\frac{1}{2}af^4\Big\{g_j^2\sum_b \mathrm{Tr}\,[(Q_j^b\Sigma)(Q_j^b\Sigma)^*] + g_j'^2\mathrm{Tr}\,[(Y_j\Sigma)(Y_j\Sigma)^*]\Big\}$$

$$\begin{aligned} V_{\rm eff}(H,\Phi) &= \frac{a}{2}(g_1^2+g_1'^2)\Big[f^2\mathrm{Tr}\,(\Phi^\dagger\Phi) - \frac{if}{2}(H\Phi^\dagger H^T - H^*\Phi H^\dagger) + \frac{1}{4}(HH^\dagger)^2 + \cdots\Big] \\ &+ \frac{a}{2}(g_2^2+g_2'^2)\Big[f^2\mathrm{Tr}\,(\Phi^\dagger\Phi) + \frac{if}{2}(H\Phi^\dagger H^T - H^*\Phi H^\dagger) + \frac{1}{4}(HH^\dagger)^2 + \cdots\Big]. \end{aligned}$$

$V_{\rm eff}$ is invariant under $SU(3)_{\rm upper} \leftrightarrow SU(3)_{\rm lower}$!

$$V_{\rm eff}(H,\Phi) = 2a(g^2+g'^2)\Big[f^2\mathrm{Tr}\,(\Phi^\dagger\Phi) + \frac{1}{4}(HH^\dagger)^2 + \cdots\Big]$$

$(a)\ H \rightarrow H,\quad \Phi \rightarrow -\Phi \qquad (b)\ H \rightarrow H^*,\quad \Phi \rightarrow \Phi^\dagger$

T-Parity ???

Higgs potential and C-parity

$$H \longrightarrow H^*$$

$$\Phi \longrightarrow \Phi^\dagger$$

$$\Sigma(x) = \exp\{2i\Pi(x)/f\}\Sigma_0$$

$$\Pi = \begin{pmatrix} 0 & H^T/\sqrt{2} & \Phi \\ H^*/\sqrt{2} & 0 & H/\sqrt{2} \\ \Phi^\dagger & H^\dagger/\sqrt{2} & 0 \end{pmatrix}$$

$$\Sigma_{\text{upper}} \Rightarrow \begin{pmatrix} \triangle & \triangle & \triangle & \triangle & \triangle \\ \triangle & \triangle & \triangle & \triangle & \triangle \\ \triangle & \triangle & \triangle & \triangle & \triangle \\ \triangle & \triangle & \triangle & & \\ \triangle & \triangle & \triangle & & \end{pmatrix} \longleftrightarrow \Sigma_{\text{lower}} \Rightarrow \begin{pmatrix} & & \triangledown & \triangledown & \triangledown \\ & & \triangledown & \triangledown & \triangledown \\ \triangledown & \triangledown & \triangledown & \triangledown & \triangledown \\ \triangledown & \triangledown & \triangledown & \triangledown & \triangledown \\ \triangledown & \triangledown & \triangledown & \triangledown & \triangledown \end{pmatrix}$$

$SU(3)_{\text{upper}} \longleftrightarrow SU(3)_{\text{lower}}$

C-parity

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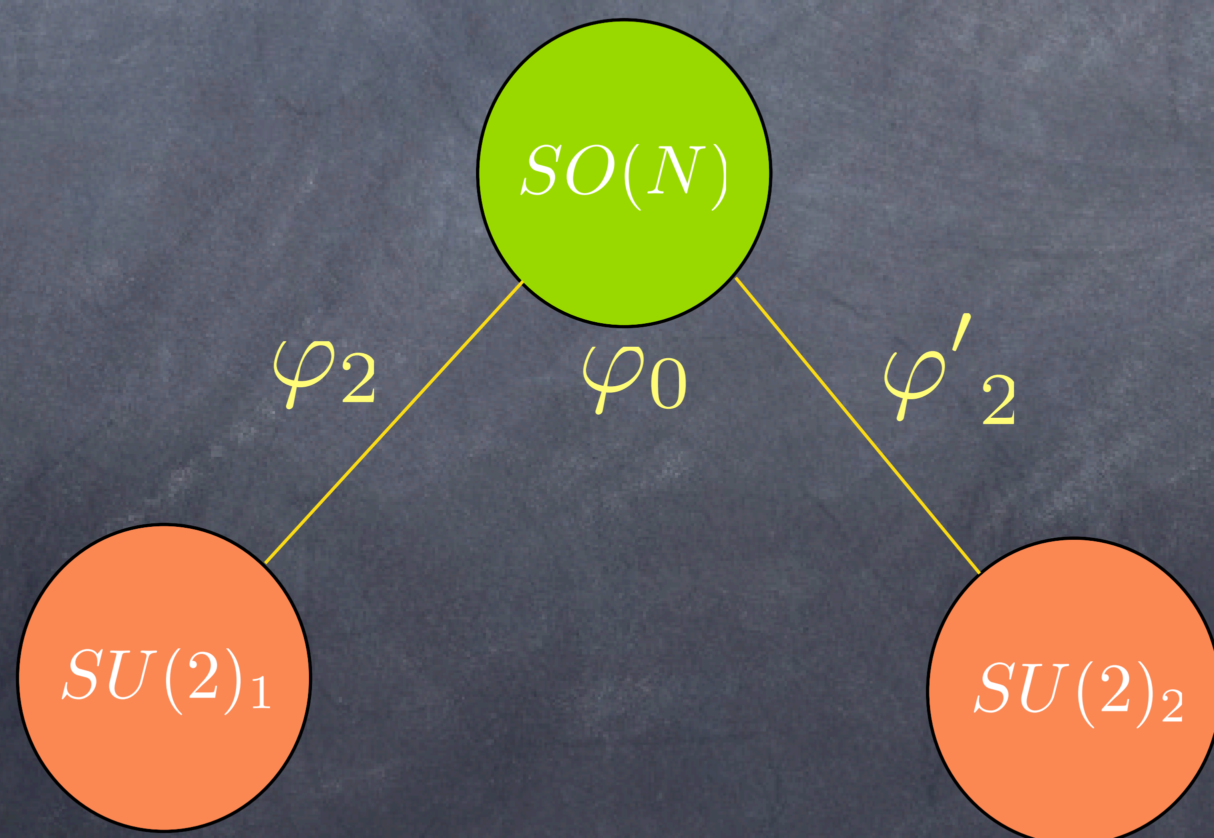
Origin of C-parity

- Question : how to realize SU(5)/SO(5) symmetry breaking pattern as well as the two global SU(3) symmetries ?
- Answer : composite Little Higgs (fermion condensation takes place through strong dynamics.)
  E.Katz, J.Y.Lee, A.Nelson, D.G.E.Walker '03
- We choose SO(N) gauge group for the technicolor-like strong interactions.
- Five Ultra fermions $\varphi_2, \varphi_2^{'}, \varphi_0$

Origin of C-parity

	$SO(N)$	$SU(2)_1$	$U(1)_1$	$SU(2)_2$	$U(1)_2$
φ_2	N	$\square$	$1/4$	$\cdot$	$1/4$
φ'_2	N	$\cdot$	$-1/4$	$\square$	$-1/4$
φ_0	N	$\cdot$	0	$\cdot$	0



Origin of C-parity

- The bilinears of Ultra-fermions correspond to the nonlinear sigma field

$$\begin{pmatrix} \langle\varphi_2\varphi_2\rangle & \langle\varphi_2\varphi_0\rangle & \langle\varphi_2\varphi_2'\rangle \\ \langle\varphi_2\varphi_0\rangle & \langle\varphi_0\varphi_0\rangle & \langle\varphi_2'\varphi_0\rangle \\ \langle\varphi_2\varphi_2'\rangle & \langle\varphi_2'\varphi_0\rangle & \langle\varphi_2'\varphi_2'\rangle \end{pmatrix} = f \begin{pmatrix} 0 & 0 & 1_2 \\ 0 & 1 & 0 \\ 1_2 & 0 & 0 \end{pmatrix}$$

$SU(3)_{\text{upper}}$ acts on φ_2 and φ_0.

$SU(3)_{\text{lower}}$ acts on φ_2' and φ_0.

$$\begin{pmatrix} \Phi & H^T/\sqrt{2} & G \\ H/\sqrt{2} & G & H^*/\sqrt{2} \\ G & H^\dagger/\sqrt{2} & \Phi^\dagger \end{pmatrix} = \Pi\Sigma_0$$

Origin of C-parity

$$f\,(\bar{Q}_1)_i\Sigma_{jx}\Sigma_{ky}u_{3R} \qquad f\,(\bar{Q}_2\Sigma_0)_i\tilde{\Sigma}_{jx}\tilde{\Sigma}_{ky}u_{3R}$$

$$f\,(\Box,\cdot)(\Box,\Box)(\cdot,\Box)(\cdot,\cdot) \qquad f\,(\cdot,\Box)(\Box,\Box)(\Box,\cdot)(\cdot,\cdot)$$

$$1 \quad \frac{H}{f} \qquad\qquad 1 \quad \frac{H}{f}$$

- Top quark (T-even) acquires a mass.
  - $(\Box,\cdot) - (\cdot,\Box)$ $\quad Mass \sim \lambda\langle H\rangle = \lambda v_{EW}$
- how about T-odd particles?
  - $(\Box,\cdot) + (\cdot,\Box)$ $\quad Mass \sim f$

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Gauge Anomaly and UV completions

Yukawa term for T-odd heavy fermion

$$f\ [(\square,\cdot)+(\cdot,\square)](\square,\square)[(\square,\cdot)+(\cdot,\square)]$$

T-odd $\quad 1+\cdots \quad$ mirror fermions

$$\frac{\kappa f}{\sqrt{2}}(\bar{\Psi}_2\xi\Psi'+\bar{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\Psi')\rightarrow\frac{\kappa f}{2}(\bar{\Psi}_2\xi\Psi'+\bar{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\Psi'+\bar{\Psi}_2\xi\Psi^{''}+\bar{\Psi}_1\Sigma_0\Omega\xi^\dagger\Omega\Psi^{''})$$

$$\Psi'=\begin{pmatrix}\psi\\ \chi\\ \tilde{\psi}\end{pmatrix}\rightarrow\Psi^{''}=-\begin{pmatrix}\tilde{\psi}\\ \chi\\ \psi\end{pmatrix}$$

- The Yukawa interaction in the Littlest Higgs model with C-parity is different with that of T-parity.
- The other linear combination of mirror fermions acquire masses in some ways.

Gauge anomaly and UV completions

- Toward a UV complete theory we probe the matter contents which are free of gauge anomalies.
- A pair of [SU(2)xU(1)]
- Due to C-parity the charges of quark (lepton) are assigned in such a way that $U(1)_1$ quantum number of ψ_1 is identical to $U(1)_2$ quantum number of ψ_2 or vice versa.
- 8 conditions for the gauge anomalies, $SU(2)_{1,2}\, SU(2)_{1,2}\, U(1)_{1,2}$, $U(1)_{1,2}\, U(1)_{1,2}\, U(1)_{1,2}$

Gauge anomaly and UV completions

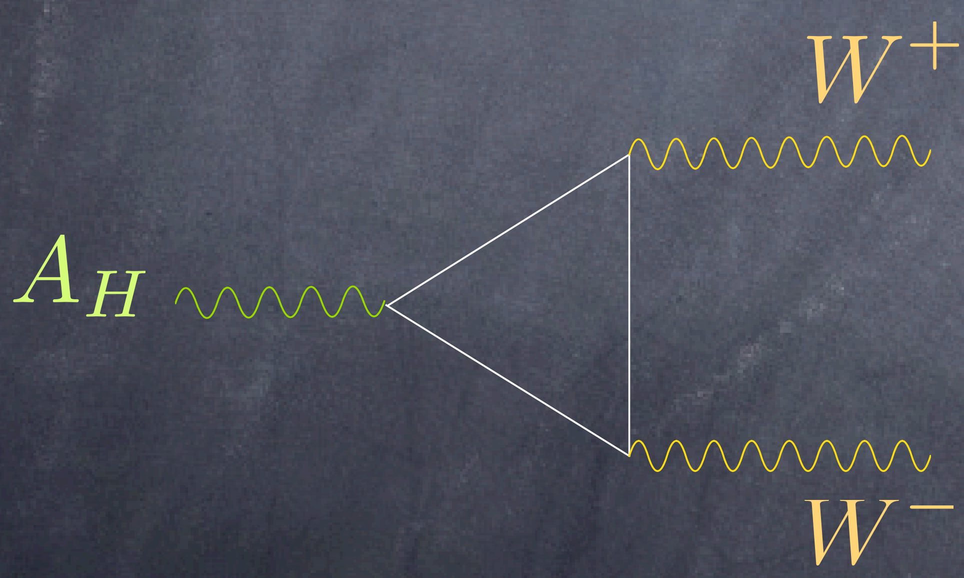
	$SU(2)_1$	$U(1)_1$	$SU(2)_2$	$U(1)_2$
q_1	**2**	1/12	**1**	1/12
t'_1	**1**	1/3	**1**	1/3
t'_{1R}	**1**	1/3	**1**	1/3
u_{3R}	**1**	1/3	**1**	1/3
l_1	**2**	-1/4	**1**	-1/4
e_R	**1**	-1/2	**1**	-1/2

	$SU(2)_1$	$U(1)_1$	$SU(2)_2$	$U(1)_2$
q_2	**1**	1/12	**2**	1/12
t'_2	**1**	1/3	**1**	1/3
t'_{2R}	**1**	1/3	**1**	1/3
d_R	**1**	-1/6	**1**	-1/6
l_2	**1**	-1/4	**2**	-1/4

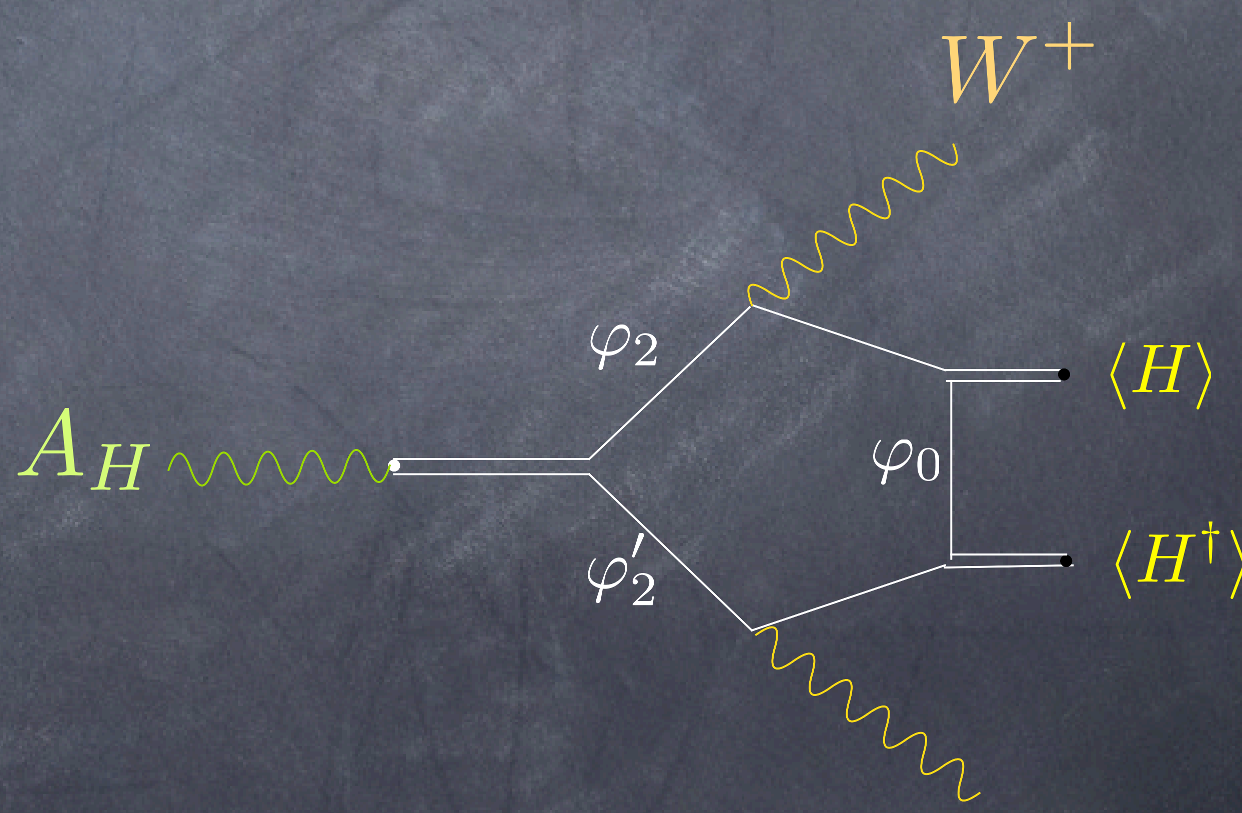
- Add Ultra fermions, mirror fermions, and more to the matter contents to make the UV theory free of gauge anomalies.
- We are still searching for the full matter contents.

Gauge anomaly and UV completions

- T-parity violation C.T. Hill and R. J. Hill '07
  - WZW term $A_H \rightarrow W^+W^- \quad ZZ$



Summed up to be zero.



Not vanish.

Summary and outlook

- Little Higgs theories are based on collective symmetry breaking.
- C-parity is just an interchange of collective symmetries and is a natural explanation of Z_2 symmetry in the Little Higgs theories.
- C-parity is a helpful tool to explore a UV complete theory of Little Higgs theories.
- T-parity violation is in general typical in the Littlest Higgs model with C-parity. Is T-parity a bogus?
- Phenomenology or UV completions are to be studied.

The End

(Thank you!)