

**On Vector Unparticle Physics from  
B to K ( $\gamma$ ) + missing energy  
(work in progress)**

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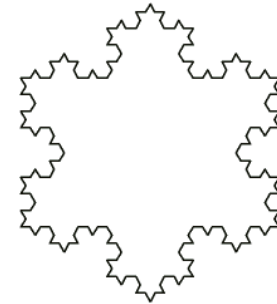
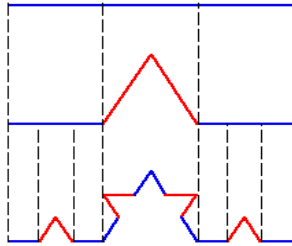
**PHENO 08 Symposium, Wisconsin-Madison**

# Outline

- Brief Introduction
- Vector Unparticle (Grinstein et.al)
- $b$  to  $s$  transition and Vector Unparticle effective interactions
- B decay into  $K$  + missing energy
- B decay into  $\gamma$  + missing energy
- Invisible Z decay
- conclusion

# Brief Introduction

Unparticles have fractional scaling dimension  
Fractional dim. appears in physics in the context of  
Dim. Reg.  
It is common in math, Fractals



# Brief Introduction

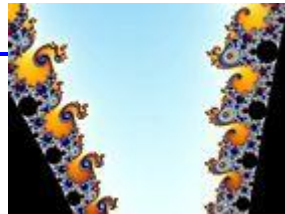
The magnification of the last image relative to the first one is about 60,000,000,000 to 1



Start



Step 1



Step 2



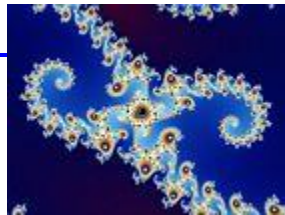
Step 3



Step 4



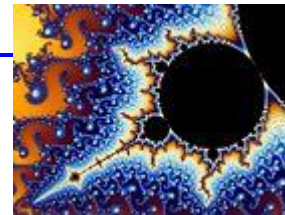
Step 5



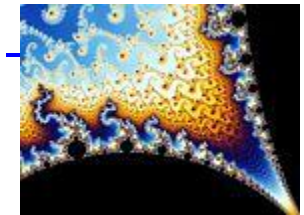
Step 6



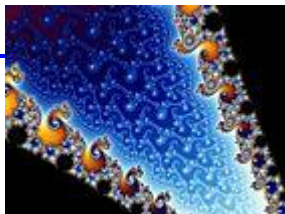
Step 7



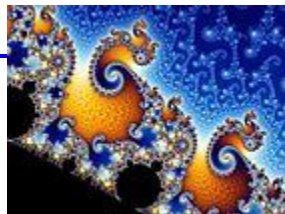
Step 8



Step 9



Step 10



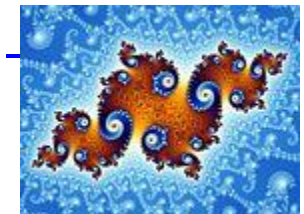
Step 11



Step 12



Step 13



Step 14

# Vector Unparticles

There are unitarity constraints on gauge invariant primary CFT operator of scaling dimension,  $d$

$$d \geq j_1 + j_2 + 2 - \delta_{j_1 j_2, 0}$$

where  $j$ 's are Lorentz spins

G. Mack, "All Unitary Ray Representations Of The Conformal Group  $SU(2,2)$  With Positive Energy," Commun. Math. Phys. **55**, 1 (1977).

For gauge invariant primary vector operator  $\mathcal{U}^\mu$   
we have  $d \geq 3$  with  $d = 3$  IFF  $\partial_\mu \mathcal{U}^\mu = 0$

B. Grinstein, K. Intriligator and I. Z. Rothstein, arXiv:0801.1140 [hep-ph].

H. Osborn and A. C. Petkou Annals Phys. **231**, 311 (1994) [arXiv:hep-th/9307010].

# Vector Unparticles

The 2-point function for  $\mathcal{U}^\mu$  is determined by

$$\langle \mathcal{O}_\mu(x)^\dagger \mathcal{O}_\nu(0) \rangle = C \frac{1}{(2\pi)^2} \frac{I_{\mu\nu}(x)}{(x^2)^d} \quad \text{where} \quad I_{\mu\nu} \equiv g_{\mu\nu} - 2 \frac{x_\mu x_\nu}{x^2}$$

and going to momentum space then

$$\frac{1}{(2\pi)^2} \frac{I_{\mu\nu}(x)}{(x^2)^d} = \frac{(d-1)\Gamma(2-d)}{4^{d-1}\Gamma(d+1)} \int \frac{d^4 k}{(2\pi)^4} e^{ik \cdot x} (k^2)^{d-2} \left[ g_{\mu\nu} - \frac{2(d-2)}{d-1} \frac{k_\mu k_\nu}{k^2} \right]$$

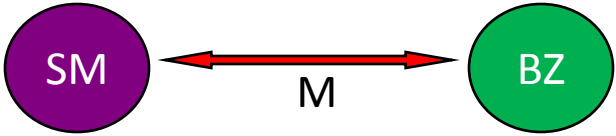
Hence,

$$\int d^4 x e^{-ik \cdot x} \langle 0 | T(O_\mu(x) O_\nu(0)) | 0 \rangle = -iC (-k^2 - i\epsilon)^{d-3} \left[ k^2 g_{\mu\nu} - \frac{2(d-2)}{d-1} k_\mu k_\nu \right]$$

$$\sum \eta^\mu \eta^\nu = -g^{\mu\nu} + a \frac{q^\mu q^\nu}{q^2} \quad a = \frac{2(d-2)}{d-1}$$

# FCNC Effective Interactions

Integrating out M between Unparticles and SM

$$\mathcal{L} = \frac{g}{M^2} \frac{1}{\Lambda_{\mathcal{U}}^{d_{\mathcal{U}}-3}} \bar{s} \gamma_{\mu} (C_V + C_A \gamma_5) b \mathcal{U}^{\mu} + h.c.$$


Integrating out M between SM operators

$$\mathcal{L} = \frac{C_V^2}{M^2} (\bar{s} \gamma_{\mu} b)^2 + h.c.$$


Bounds from effective contact interactions for  $\Delta B = 2$  transitions

$$\mathcal{L}_{\Delta B=2} = \frac{C_{bd}^2}{M^2} (\bar{d}_L \gamma_{\mu} b_L)^2 + \frac{C_{bs}^2}{M^2} (\bar{s}_L \gamma_{\mu} b_L)^2$$

These terms contribute to B mass difference

$$\frac{\Delta m_B}{m_B} = \frac{2|M_{12}^B|}{m_B} \sim \frac{C^2}{3} \frac{f_B^2}{M^2}$$

# FCNC Effective Interactions

Experimentally

$$\frac{\Delta m_B}{m_B} \sim 6.3 \times 10^{-14} \quad \text{and} \quad \frac{\Delta m_{B_s}}{m_{B_s}} \sim 6.3 \times 10^{-12}$$

Hence

$$C_{bd}^2 \leq 6 \times 10^{-6} \left( \frac{M}{1 \text{ TeV}} \right)^2 \quad \text{and} \quad C_{bs}^2 \leq 2 \times 10^{-4} \left( \frac{M}{1 \text{ TeV}} \right)^2$$

In the numerical analysis we will be using

$$\left( \frac{C}{M} \right)^2 \leq 10^{-11} \text{ GeV}^{-2}$$



# But Scale Invariance is Broken



$$\frac{\Lambda^{3-d}}{M} |H|^2 \mathcal{U}$$

Shirman et. al, Phys. Rev. D76 (2007) 075004

Shirman et. al, Phys. Rev. D76 (2007) 115002

$$\mathcal{U}^2 (H^\dagger H)$$

T. Kikuchi et al arXiv:0711.1506 [hep-ph]

$$\frac{\Lambda^{6-d^*}}{M^4} |H|^2 \mathcal{U}^\mu \mathcal{U}_\mu$$

V. Barger et al Phys.Lett.B661: 276-286,2008

$$|\langle 0 | O_{\mathcal{U}} | P \rangle|^2 \rho(P^2) = A_{d_{\mathcal{U}}} \theta(P^0) \theta(P^2 - \mu^2) (P^2 - \mu^2)^{d_{\mathcal{U}} - 2}$$

We will take  $\mu = 0.5 \text{ GeV}$

# B to K + Missing energy

SM

$$B \rightarrow K + \nu\bar{\nu}$$

Effective Interaction

$$H_{eff} = C (\bar{s}b)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$C = \frac{2G_F}{\sqrt{2}} \frac{\alpha}{2\pi} V_{tb} V_{ts}^* \frac{X(x_t)}{\sin^2\theta_w}$$

Inami Lim fn.

$$X(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 1}{x_t - 1} + \text{Ln}(x_t) \frac{3x_t - 6}{(x_t - 1)^2} \right]$$

$$x_t = \frac{m_t^2}{m_W^2}$$

Theoretical estimate

$$\mathcal{BR}(B \rightarrow K\nu\bar{\nu}) \sim 10^{-6}$$

Experimental Bound

$$\mathcal{BR}(B \rightarrow K\nu\bar{\nu}) < 1.4 \times 10^{-5}$$

# B to K + Missing energy

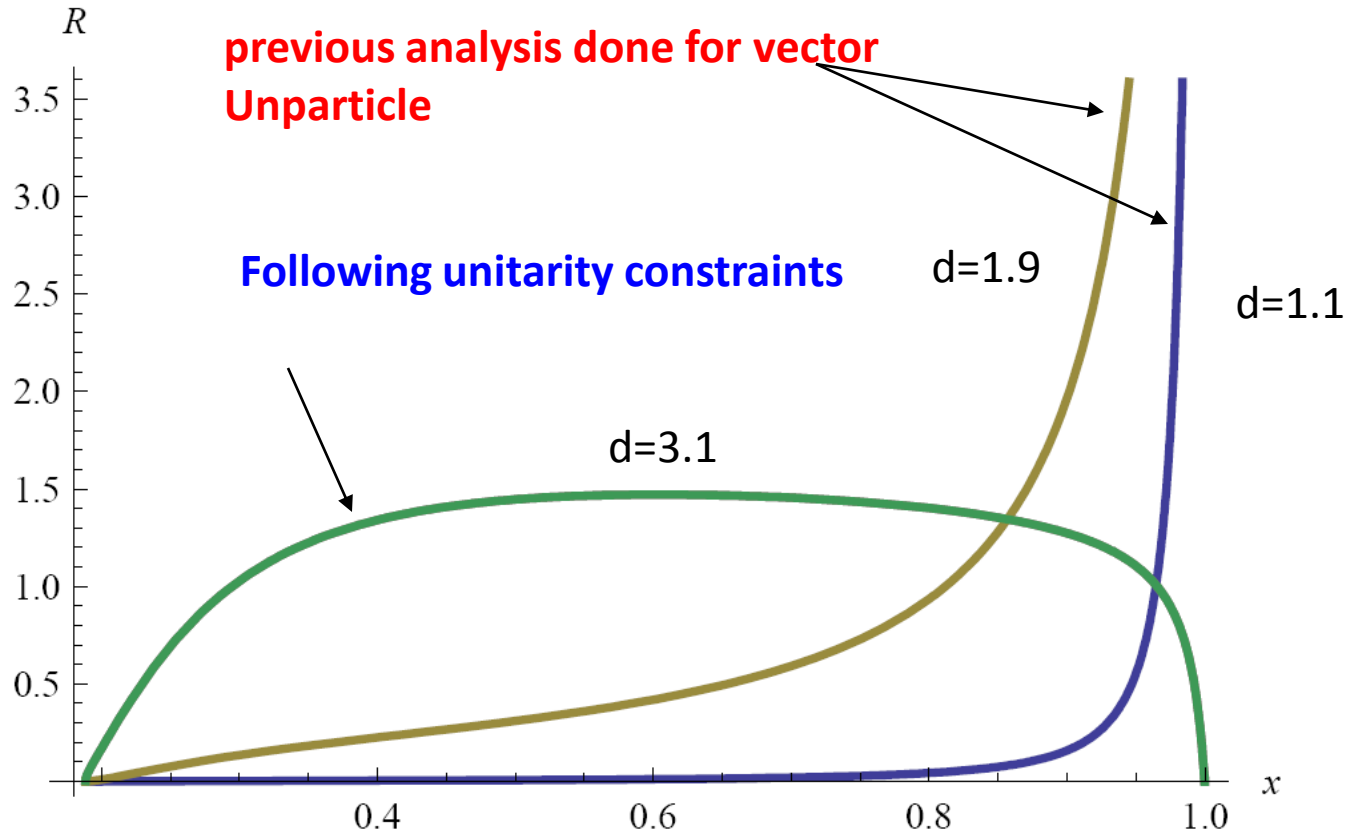
$$B \rightarrow K + \mathcal{U}$$

$$\frac{d\Gamma}{dE_X} = \frac{A_{d\mathcal{U}}}{8\pi^2 m_B} \frac{\sqrt{E_X^2 - m_X^2} \theta(m_B^2 - 2m_B E_X + m_X^2 - \mu^2)}{(m_B^2 - 2m_B E_X + m_X^2 - \mu^2)^{2-d\mathcal{U}}} |\overline{\mathcal{M}}|^2$$

$$\begin{aligned} |\overline{\mathcal{M}}|^2 &= \left(\frac{g}{M}\right)^2 \left(\frac{C_V}{M}\right)^2 \frac{1}{\Lambda^{2d-6}} (|f_+|^2 \left[ a \frac{(m_B^2 - m_K^2)^2}{m_B^2 + m_K^2 - 2m_B E_K} - (m_B^2 + m_K^2 + 2m_B E_K) \right] \\ &\quad + (a-1)|f_-|^2 (m_B^2 + m_K^2 - 2m_B E_K) \\ &\quad + 2(a-1)f_+ f_- (m_B^2 - m_K^2)) \end{aligned}$$

# B to K + Missing energy

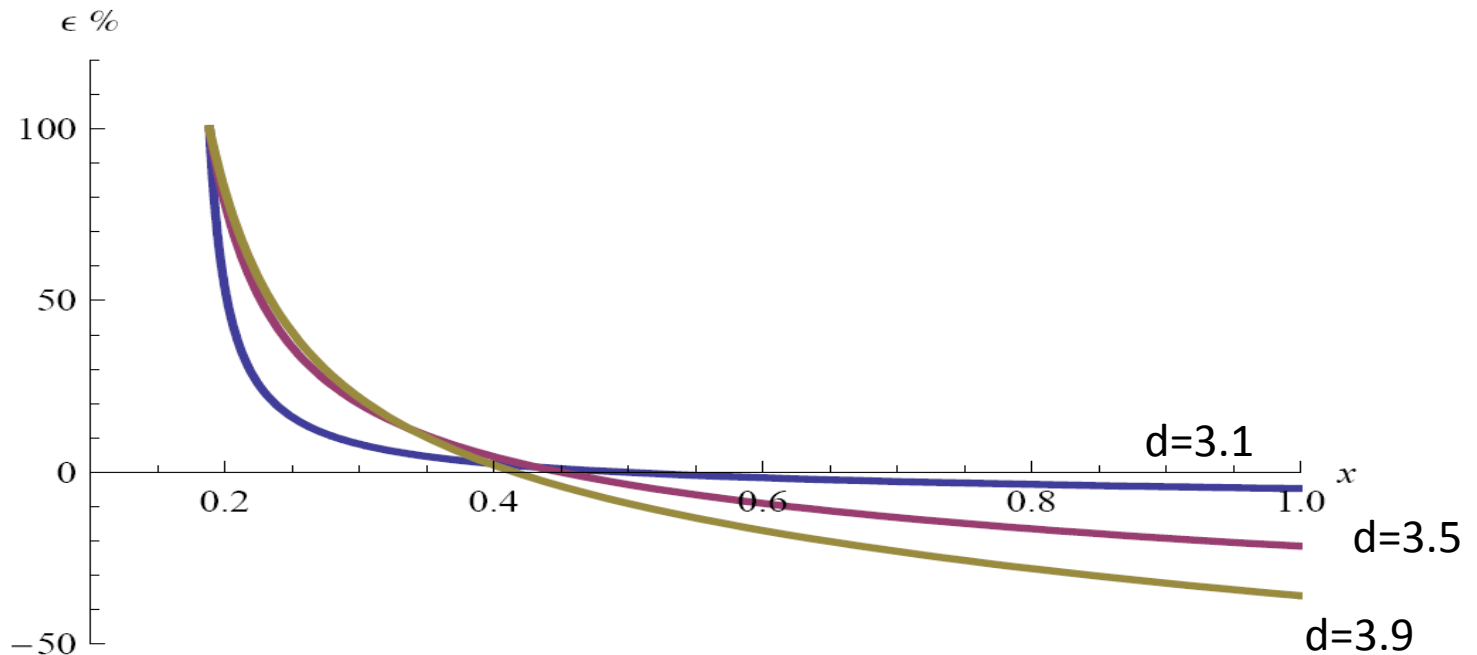
Plotting the spectrum  $R \equiv \frac{1}{\Gamma} \frac{d\Gamma}{dx}$  where  $x = \frac{2E_K}{m_B}$



# B to K + Missing energy

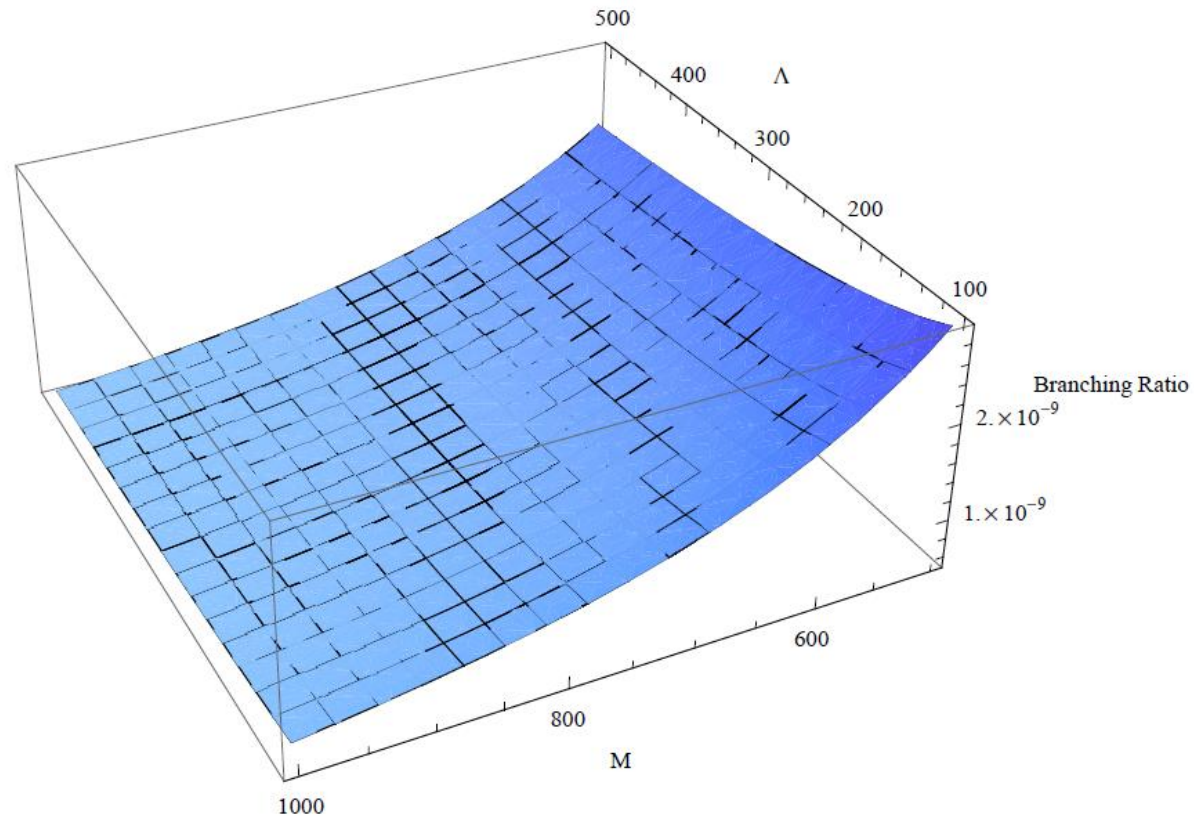
The difference between using the correct treatment of vector Unparticle and the old one can be parameterized by

$$\epsilon(x, d) \equiv \frac{R(x, d, a) - R(x, d, a = 1)}{R(x, d, a)} 100\%$$



# B to K + Vector Unparticle

Dependence of the branching ratio on M and  $\Lambda$  in GeV



# B to $\gamma$ + Missing energy

SM

$$B \rightarrow \gamma + \nu\bar{\nu}$$

Effective Interaction

$$H_{eff} = C (\bar{s}b)_{V-A} (\bar{\nu}\nu)_{V-A}$$

$$C = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2\theta_W} V_{tb}V_{tq}^* \frac{x}{8} \left( \frac{x+2}{x-1} + \frac{3x-6}{(x-1)^2} \text{Ln}(x) \right)$$

Theoretical estimate

$$\mathcal{BR}(B_d \rightarrow \gamma\nu\bar{\nu}) \sim \times 10^{-9}$$

$$\mathcal{BR}(B_s \rightarrow \gamma\nu\bar{\nu}) \sim \times 10^{-8}$$

Experimental Bound

$$\mathcal{BR}(B_d \rightarrow \gamma\nu\bar{\nu}) < 4.7 \times 10^{-5}$$

# B to $\gamma$ + Missing energy

$$B \rightarrow \gamma + \mathcal{U}$$

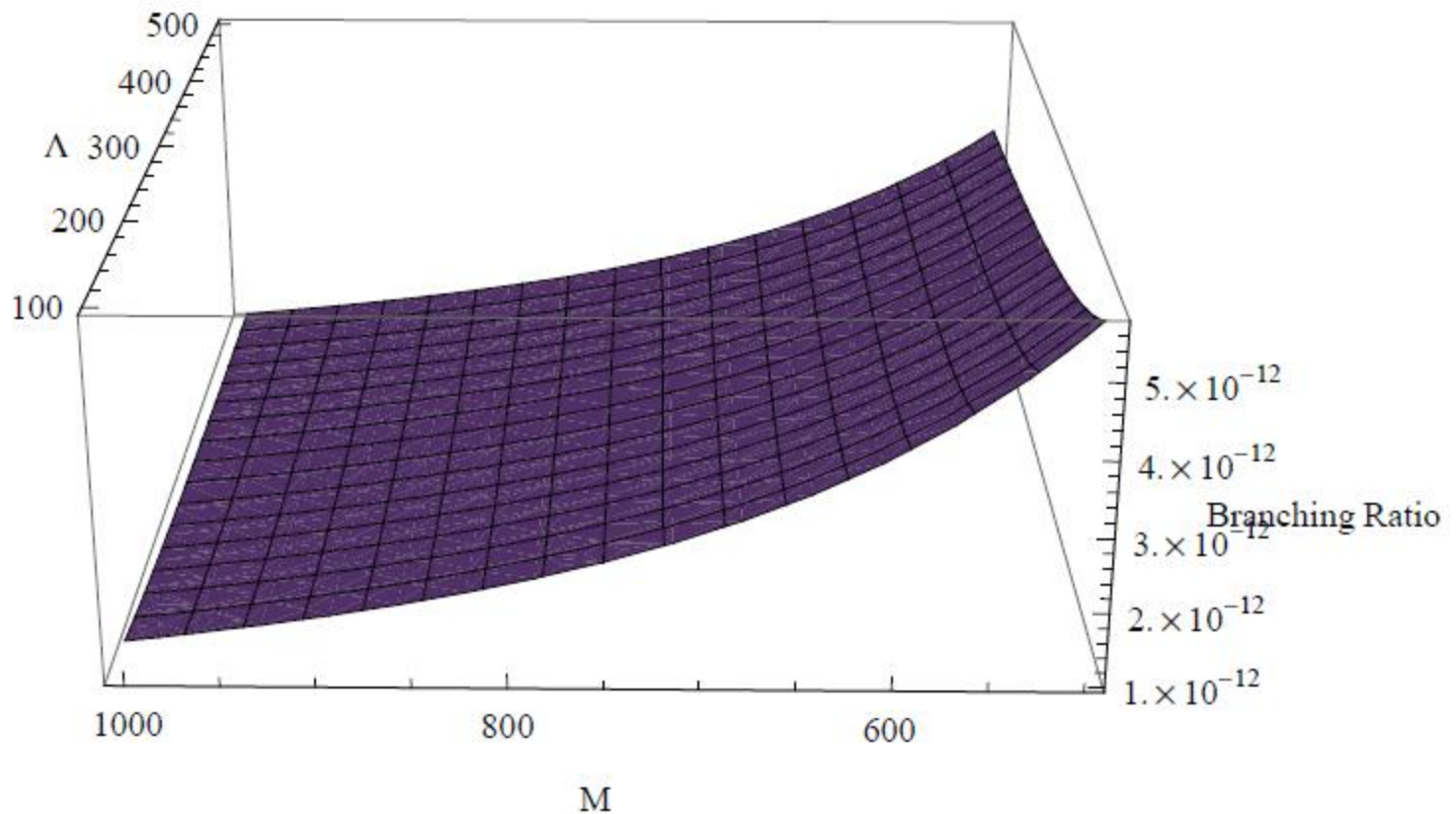
$$|\overline{\mathcal{M}}|^2 = 2(k \cdot q)^2 \left(\frac{g}{M}\right)^2 \left(\frac{C_V}{M}\right)^2 \frac{1}{\Lambda^{2d-6}} \frac{4\pi\alpha}{m_B^2} (F_V^2(q^2) + F_A^2(q^2))$$

$$\frac{d\Gamma}{dE_\gamma} = \frac{A_{d\mathcal{U}}}{8\pi^2 m_B} \frac{E_\gamma \theta(m_B^2 - 2m_B E_\gamma - \mu^2)}{(m_B^2 - 2m_B E_\gamma - \mu^2)^{2-d\mathcal{U}}} |\overline{\mathcal{M}}|^2$$



# B to $\gamma$ + vector Unparticle

Dependence of the branching ratio on M and  $\Lambda$  in GeV



# Invisible Decay of Z boson

Effective interaction

$$\mathcal{L} = \frac{\Lambda^{3-d}}{M^2} H^\dagger D_\mu H U^\mu \sim \frac{\Lambda^{3-d}}{M^2} \frac{v}{2} M_Z Z_\mu U^\mu$$

The decay rate of Z into vector Unparticle is

$$\Gamma(Z \rightarrow \mathcal{U}) = \frac{v^2 A_d}{8M_Z} \left( \frac{M_Z}{\Lambda} \right)^{2d-6} \left( \frac{M_Z}{M} \right)^4$$

Bounds on M from effective contact interactions

$$\mathcal{L} = \frac{c}{M} |H^\dagger D_\mu H|^2$$

$M > 4.6 \text{ TeV}$  for  $c = -1$       and       $M > 5.6 \text{ TeV}$  for  $c = 1$

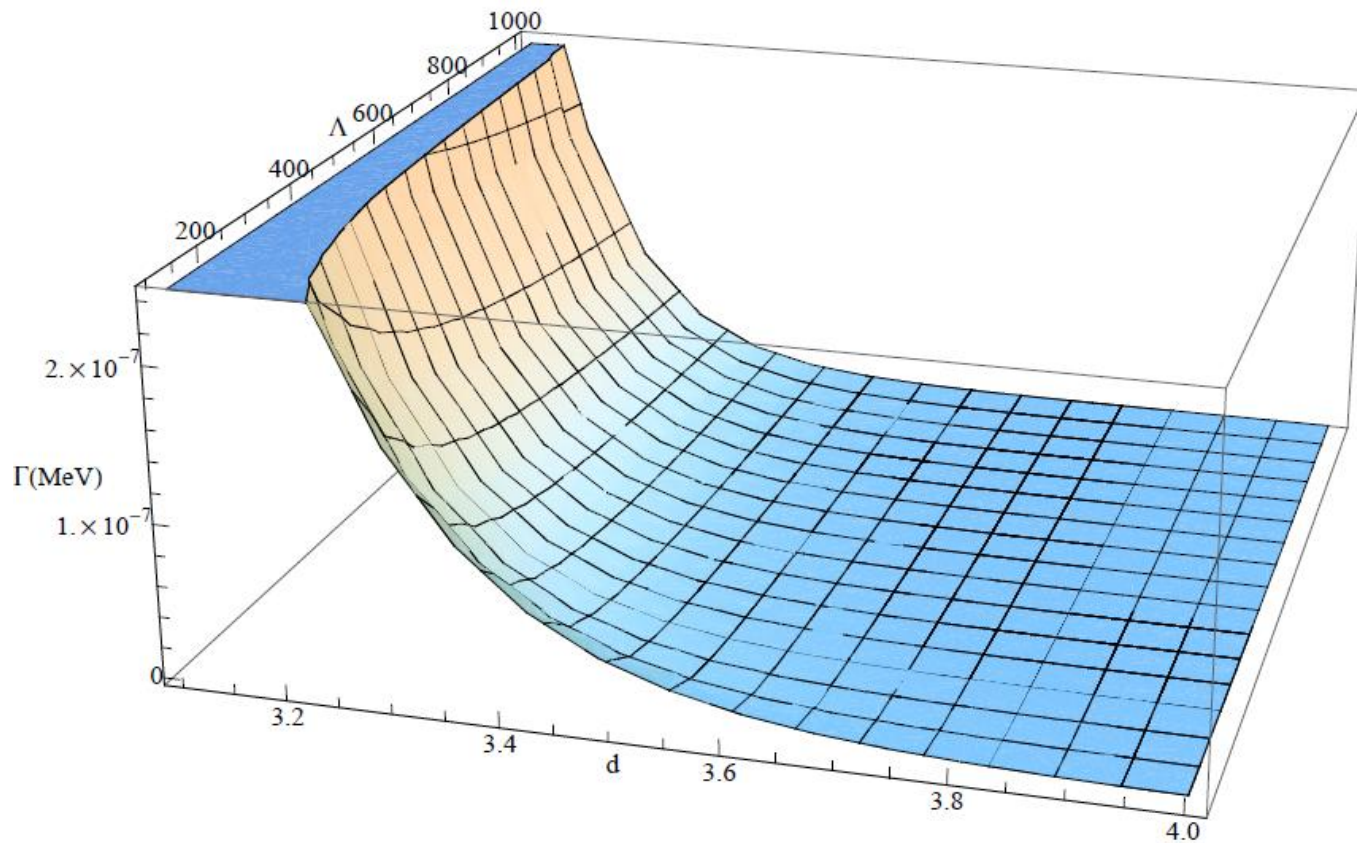
# Invisible Decay of Z boson

New physics contribution to invisible Z decay is severely constrained

$$\Gamma^{SM}(Z \rightarrow invisible) = 501.65 \pm 0.11 \text{ MeV}$$

$$\Gamma^{Exp.}(Z \rightarrow invisible) = 499.0 \pm 1.5 \text{ MeV}$$

# Invisible Decay of Z boson into Vector Unparticle



# Conclusion

- We have studied missing energy signals in the  $b$  to  $s$  FCNC transition in  $B$  to  $K$  + missing energy and  $B$  to  $\gamma$  + missing energy.
- These channels cannot be used to constrain vector Unparticle because the branching ratios are very small for reasonable values of parameter space
- High energy probes as in  $Z$  decay are also very suppressed
- Unparticles as sub classes of hidden valleys, HEIDI, Infraparticle, self interaction, gauged unparticles
- More studies are needed for models that are scale invariant but not conformal invariant