Generalizing the Method of Kinematical Endpoints

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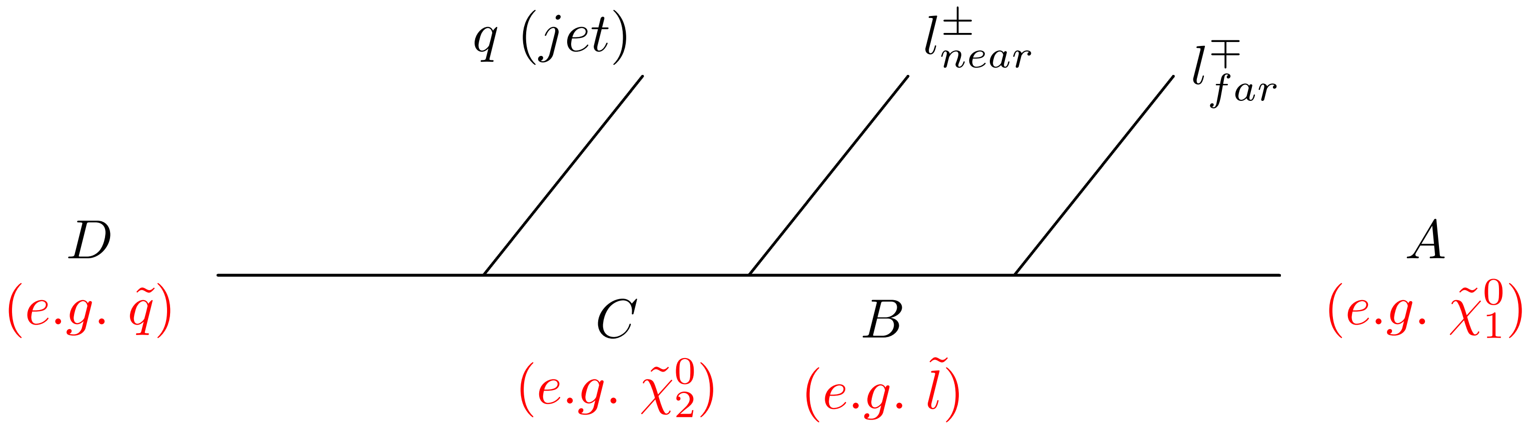
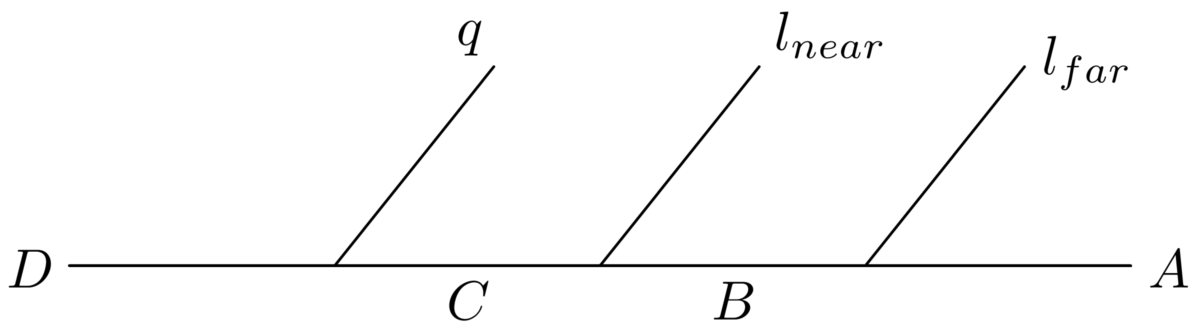
The cascade decay

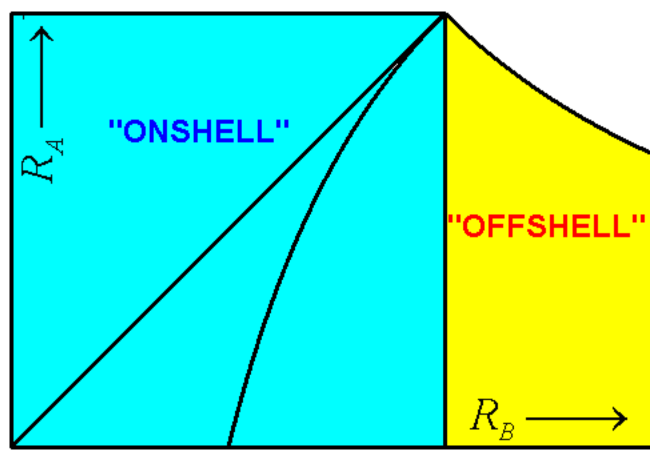


- B, C, D heavy particles; A unobservable
- observe q, l^+, l^- to determine m_A, m_B, m_C, m_D

Parameter space



$$\underbrace{R_A \equiv \left(\frac{m_A}{m_B}\right)^2 \quad R_B \equiv \left(\frac{m_B}{m_C}\right)^2}_{\text{categorizes single-lepton distributions}} \qquad \underbrace{R_C \equiv \left(\frac{m_C}{m_D}\right)^2}_{\text{categorizes di-lepton distributions}} \qquad \underbrace{(m_D)^2}_{\text{overall scale}}$$

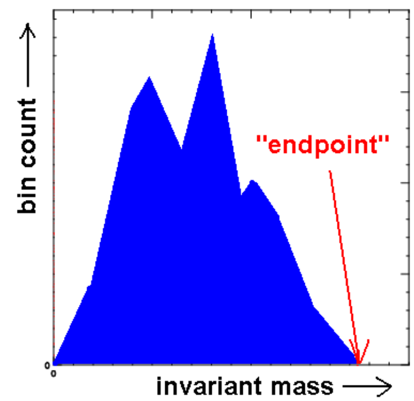


constant R_C slice

The endpoint method

- $p_D = ?$, $p_A = ?$ $\longrightarrow$ cannot use resonances

- single invariant mass distributions (histograms): m_{ll}, m_{qll}, m_{ql}'s

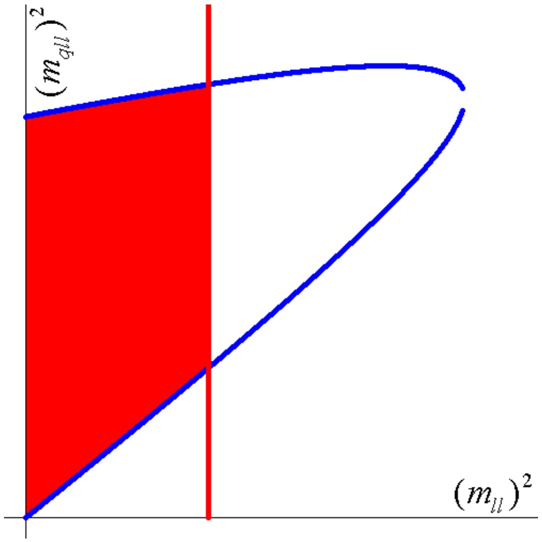


- $m_{ll}^{\max}$, $m_{qll}^{\max}$, $m_{ql}^{\max}$'s $\longrightarrow$ R_A, R_B, R_C, m_D

- *DUPLICATE SOLUTIONS CAN OCCUR*

- use 2-variable distributions (scatter plots) to resolve the ambiguity
  – more features: endpoints extended to boundary lines

$(m_{qll})^2$ vs. $(m_{ll})^2$ boundary lines

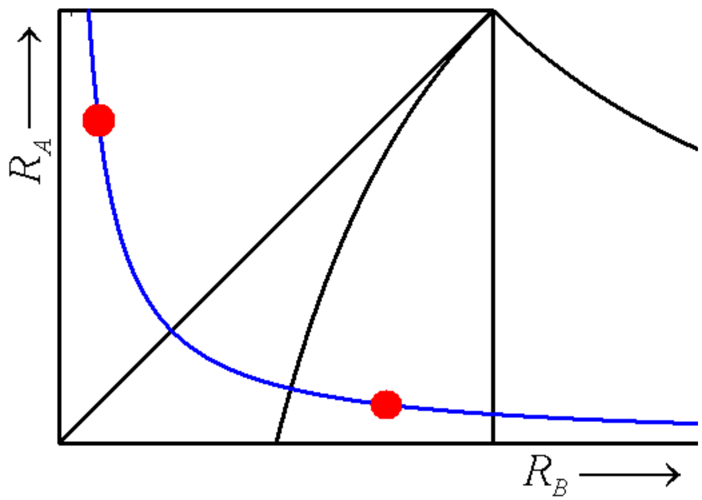


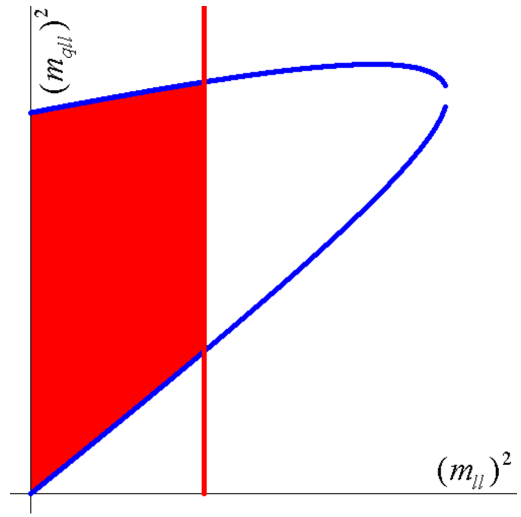
- vertical boundary is $(m_{ll}^{\max})^2$
- $(m_{qll})^2$ intercept is always available
- intersections of vertical boundary with curved boundary provide two more equations

$$\left(\frac{m_{ll}}{m_D}\right)^2 = R_C(1-R_A)(1-R_B)$$

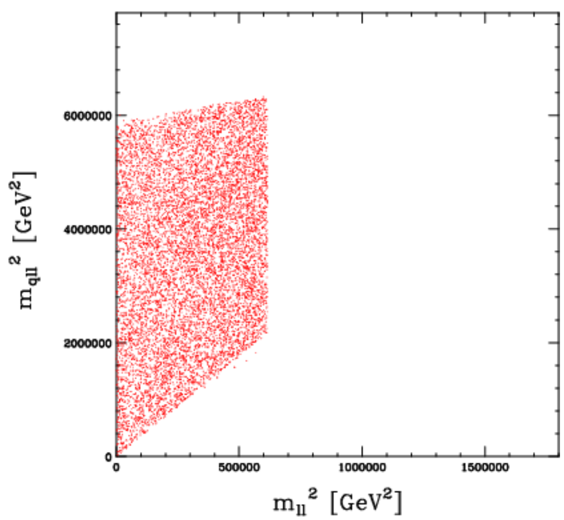
$$\left(\frac{m_{qll}}{m_D}\right)^2 = \frac{1}{2}(1-R_C)(1-R_A R_B) + \frac{1+R_C}{2R_C}\left(\frac{m_{ll}}{m_D}\right)^2$$
$$\pm\frac{1-R_C}{2R_C}\sqrt{\Big(R_C(1-R_A R_B)\Big)^2 - 2R_C(1+R_A R_B)\left(\frac{m_{ll}}{m_D}\right)^2 + \left(\frac{m_{ll}}{m_D}\right)^4}$$

$(m_{qll})^2$ vs. $(m_{ll})^2$ phase space simulation

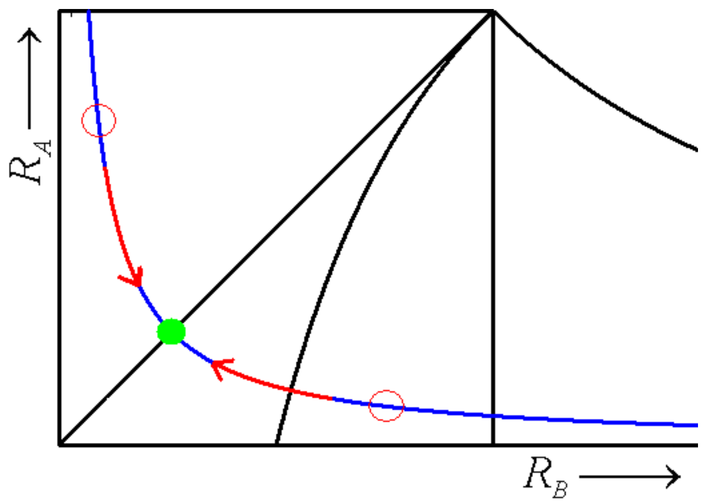


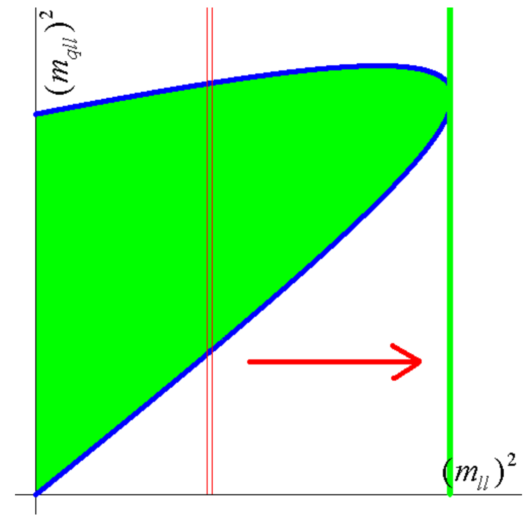


- simulation confirms boundary lines
- curved boundary same for every parameter point on hyperbola
- exchanging $R_A \leftrightarrow R_B$ gives same distribution . . . duplication remains



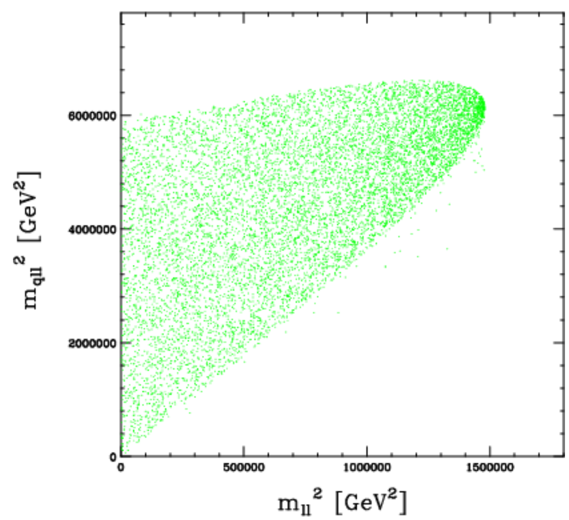
$(m_{qll})^2$ vs. $(m_{ll})^2$ phase space simulation



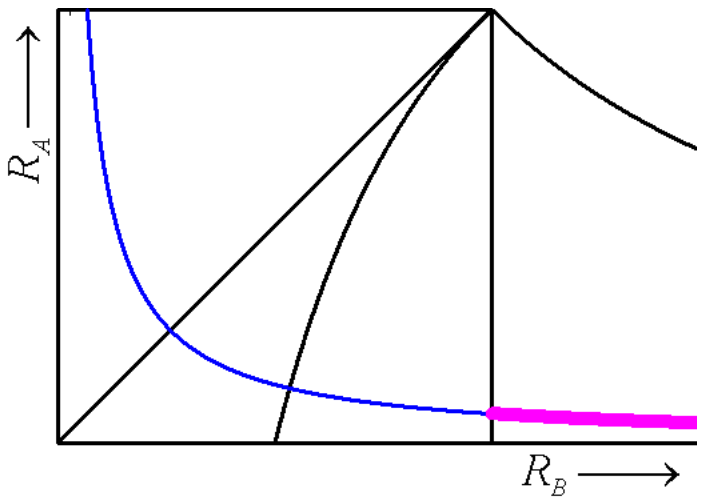


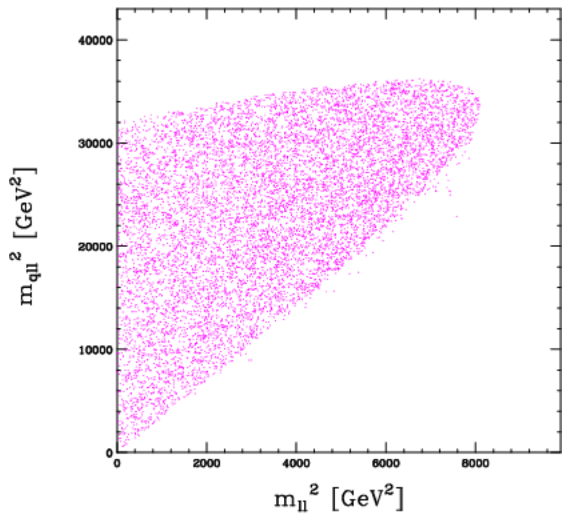
- If parameter point slides along hyperbola to $R_A = R_B$ . . .

- . . . then vertical boundary slides toward apex of curved boundary



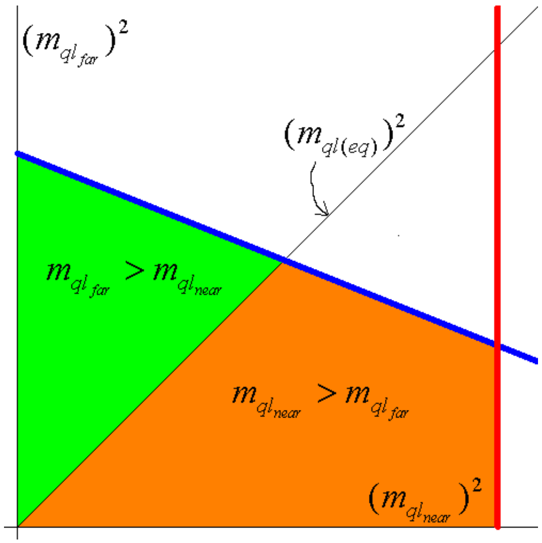
$(m_{qll})^2$ vs. $(m_{ll})^2$ phase space simulation





- In the "offshell" region of parameter space, there is no vertical boundary.
- In fact, in the "offshell" region of parameter space, neither R_A nor R_B can, in principle, be determined based on kinematics alone; only the product $R_A R_B$ can be determined based on kinematics.

$(m_{ql_{far}})^2$ vs. $(m_{ql_{near}})^2$ boundary lines

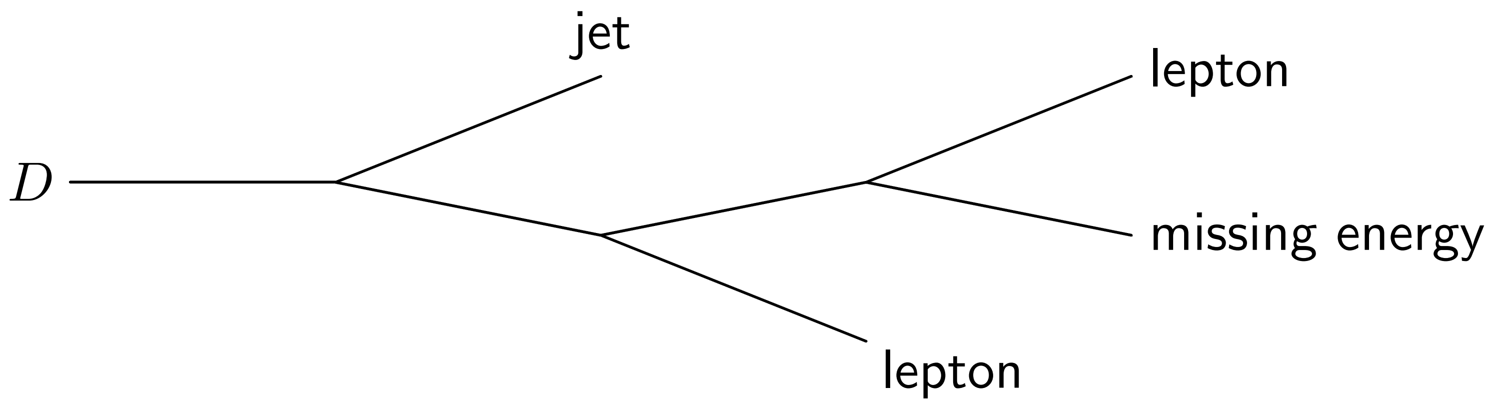


- vertical boundary is $(m^{\max}_{ql_{near}})^2$
- negatively sloped upper boundary is $(m^{\max}_{ql_{far}})^2$ given $(m^{\max}_{ql_{near}})^2$
- distribution assumed to be unobservable
- plot divided into two areas based on which m_{ql} is larger

$$\left(\frac{m_{ql_{near}}}{m_D}\right)^2 = (1 - R_B)(1 - R_C)$$

$$\left(\frac{m_{ql_{far}}}{m_D}\right)^2 = (1 - R_A)(1 - R_C) - (1 - R_A)\left(\frac{m_{ql_{near}}}{m_D}\right)^2$$

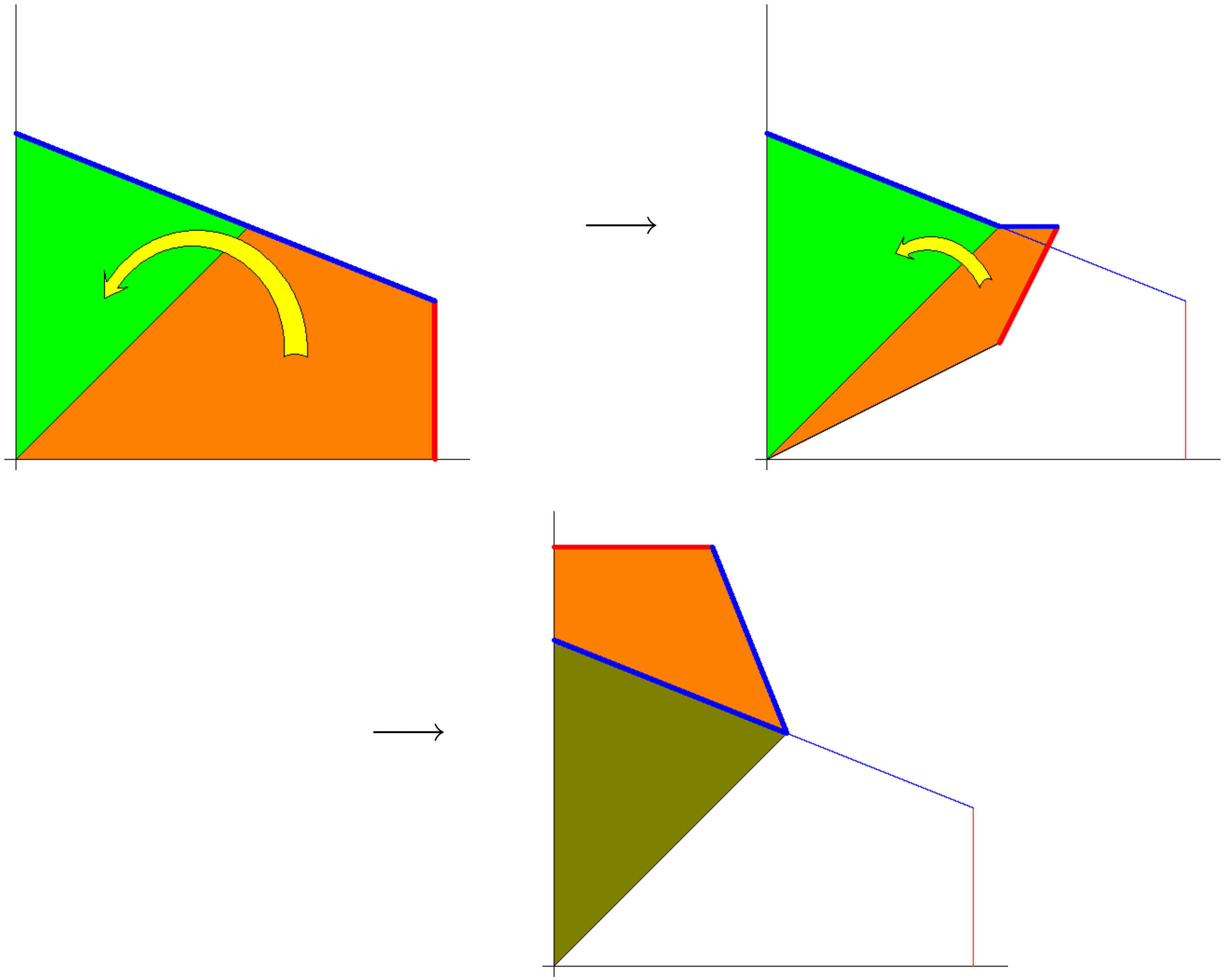
$$\{m_{ql_{near}}, m_{ql_{far}}\} \rightarrow \{m_{ql(low)}, m_{ql(high)}\}$$



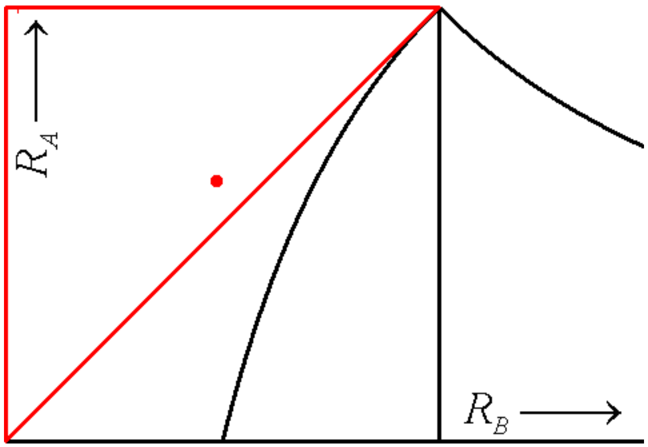
$$m_{ql(low)} \equiv \min[m_{ql_{near}}, m_{ql_{far}}]$$

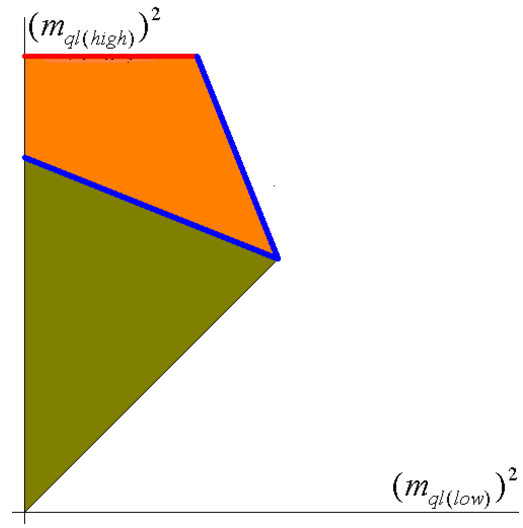
$$m_{ql(high)} \equiv \max[m_{ql_{near}}, m_{ql_{far}}]$$

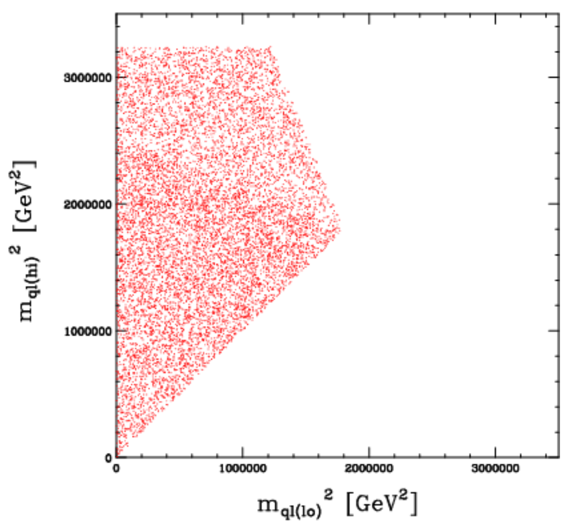
Folding across $(m_{ql(eq)})^2$



Shapes of $(m_{ql(high)})^2$ vs. $(m_{ql(low)})^2$

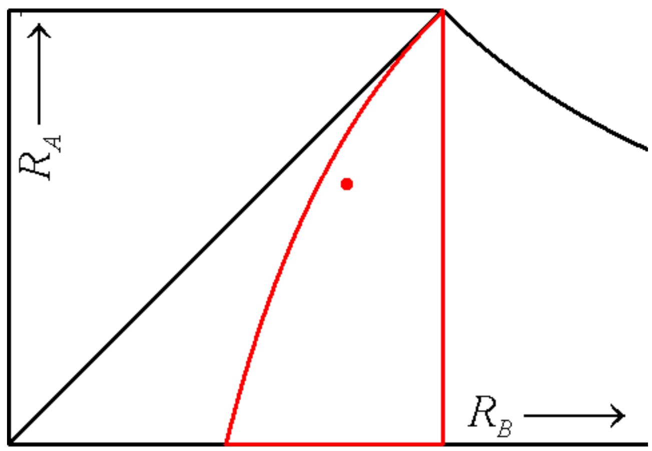


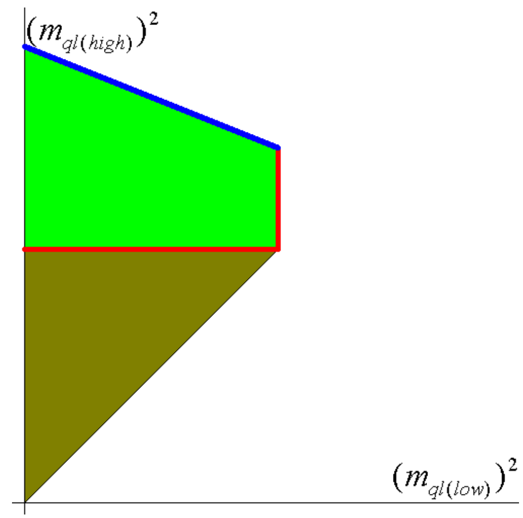




- simulation confirms boundary lines
- featureless horizontal upper boundary
  $\Rightarrow R_A > R_B$

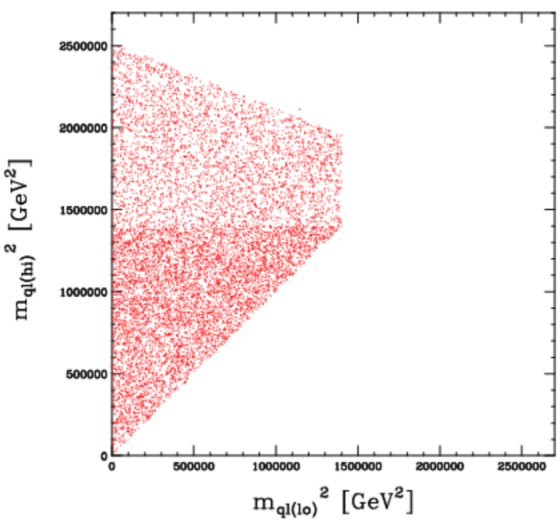
Shapes of $(m_{ql(high)})^2$ vs. $(m_{ql(low)})^2$



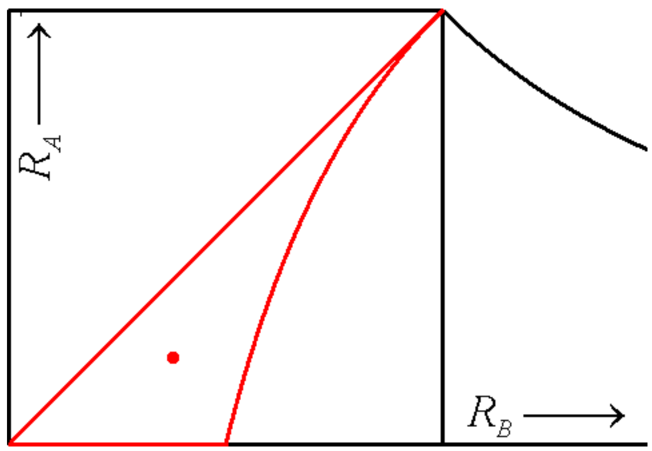


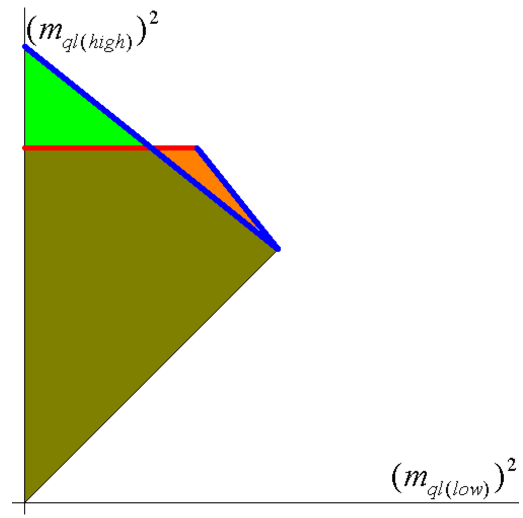
- simulation confirms boundary lines

- featureless negatively sloped upper boundary
  $\Rightarrow R_B > R_A$



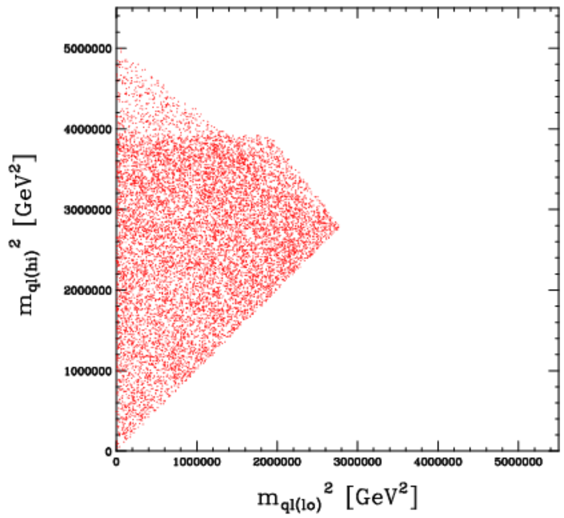
Shapes of $(m_{ql(high)})^2$ vs. $(m_{ql(low)})^2$



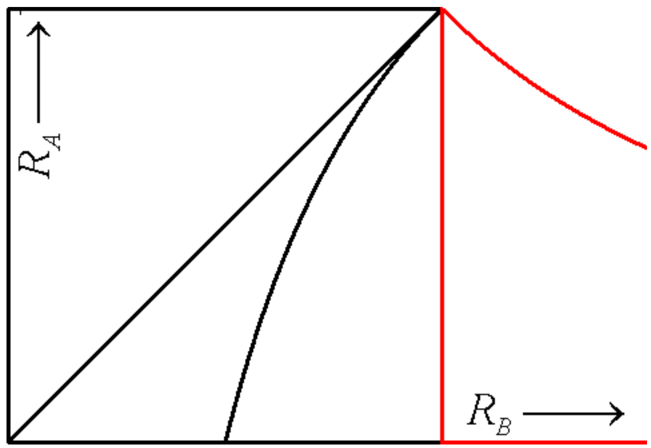


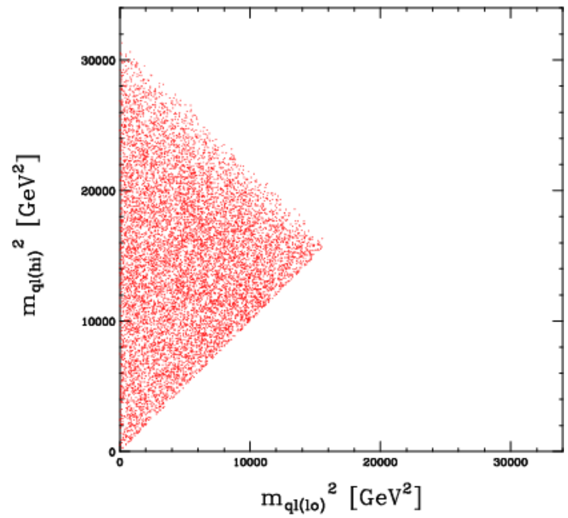
- simulation confirms boundary lines

- featured upper boundary
  $\Rightarrow R_B > R_A$



Shapes of $(m_{ql(high)})^2$ vs. $(m_{ql(low)})^2$

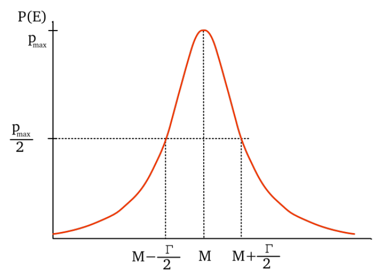




- In the "offshell" region of parameter space the simulation exhibits no features.

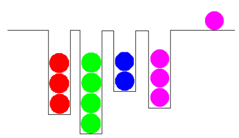
Experimental considerations

- finite width

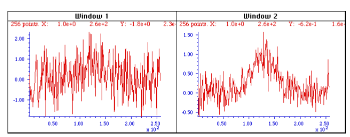


- detector effects

- combinatorics



- backgrounds



- spin

Conclusion

- The unknown masses in the cascade decay cannot always be determined from the endpoints of the invariant mass distributions alone.

- Additional features in the invariant mass distributions can be recognized, and the 2-variable distributions exhibit these features in a straightforward way.

- We do not yet know how these features can be extracted from realistic data.