Polynomials for the determination of masses in events with missing energy

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Motivation

If our current particle picture of Dark Matter is correct, the LHC is likely to be a Dark Matter factory. Realistic models containing a Dark Matter particle tend to be very similar.

- A symmetry is added to keep Dark Matter stable $\rightarrow$ Dark Matter is produced in pairs.

- Symmetries which keep Dark Matter stable are often taken from other sources (because we prefer as simple a model as possible), such as:

  - Proton Stability (R-Parity in SUSY)

  - Custodial Symmetry (solving Little Hierarchy Problem)

  - 5D momentum conservation (KK number conservation in UED)

  "Other Sources" for the symmetry generically means "Other Particles".

Cross Sections as Probability Densities:

What are we actually measuring? Let us define a probability distribution for an event. A cross section generally is given by

$$\sigma = \frac{1}{F}\int \left|\mathcal{M}(p_0^\mu, p_i^\mu, \lambda)\right|^2 \left(\prod_i \frac{d^3\vec{p}_i}{(2\pi)^3 2E_i}\right)(2\pi)^4\delta^4(p_0^\mu - \sum_i p_i^\mu)$$

for some initial state momenta p_0^μ and final state momenta p_i^μ. This is a zero-dimensional projection of a high-dimensional phase space, and contains very little information! Buried in here somewhere is all the information that is to be had. Let us do a little rearrangement to retain all information in the high-dimensional space.

$$P(\vec{p}_1, \ldots, \vec{p}_N) = \frac{1}{\sigma}\frac{d\sigma}{\prod_i d^3\vec{p}} = \frac{(2\pi)^{4-3N}}{2^N F\sigma \prod_i E_i}\left|\mathcal{M}(p_0^\mu, p_i^\mu, \lambda)\right|^2 \delta^4\left(p_0^\mu - \sum_i p_i^\mu\right).$$

this is a *probability density* expressing the probability of a particular configuration of momenta. For N external particles, it is a $3N-4$ dimensional space.

Polynomial Systems

$$P(p_i^\mu|\lambda) = \frac{1}{\sigma}\frac{d\sigma}{\prod_i d^3\vec{p}} = \frac{(2\pi)^{4-3N}}{2^N F\sigma \prod_i E_i}\left|\mathcal{M}(p_0^\mu, p_i^\mu, \lambda)\right|^2 \delta^4\left(p_0^\mu - \textstyle\sum_i p_i^\mu\right). \quad (1)$$

In principle, one could directly compare this PDF (*Probability* Density Function) between simulated events and data. But, high-dimensional spaces require a lot of data to map out.

- Project onto lower dimensional space (e.g. Breit-Wigner, end-point/edge techniques)

- Use a Likelihood or "Matrix Element" method

The Neyman-Pearson lemma tells us that the most powerful statistic for differentiating two hypotheses λ and λ' is the ratio of two Likelihoods. Our Likelihood is

$$L(\lambda|\{p_i^\mu\}) = \prod_{i=1}^{N} P(p_i^\mu|\lambda).$$

Polynomial Systems

In a Collider with missing energy, the PDF is defined as

$$P(p_i^\mu|\lambda) = \int \frac{(2\pi)^{4-3N}}{2^N F\sigma \prod_i E_i} \left|\mathcal{M}(p_0^\mu, p_i^\mu, \lambda)\right|^2 \delta^4 \left(p_0^\mu - \sum_i p_i^\mu\right) dx_1 dx_2 d^3p_1 d^3p_2$$

Now let us go into the narrow width approximation by replacing

$$\frac{1}{(q^2 - M^2)^2 - M^2\Gamma^2/4} \rightarrow \frac{\pi}{M\Gamma}\delta(q^2 - M^2)$$

in $\left|\mathcal{M}(p_0^\mu, p_i^\mu, \lambda)\right|^2$, for some hypothesis diagram.

Alternatively, one can simply insert the appropriate delta functions corresponding to a diagram, and view this as a variable change.

Note that this integral is 4 dimensional at a hadron collider. Therefore, *by specifying 4 masses, the integral is reduced to a discrete set of solutions for the missing momenta*.

A pair of simultaneous quadratics is not guaranteed to have a solution!

Combining Events: our method

Given the decay $\tilde{q}\tilde{q} \to q\chi_2^0 q\chi_2^0 \to ql\tilde{l}ql\tilde{l} \to qll\chi_1^0 qll\chi_1^0$ (as occurs in SPS 1a):

This process is underconstrained by 2. There are 4 kinematic unknowns and 6 unknown intermediate masses. So, not enough constraints to solve simultaneously for the masses and the kinematic unknowns in one event.

But, under the assumption that the masses are the same on both sides of the event, and the same between two events, one can solve for the masses using a *pair* of events.

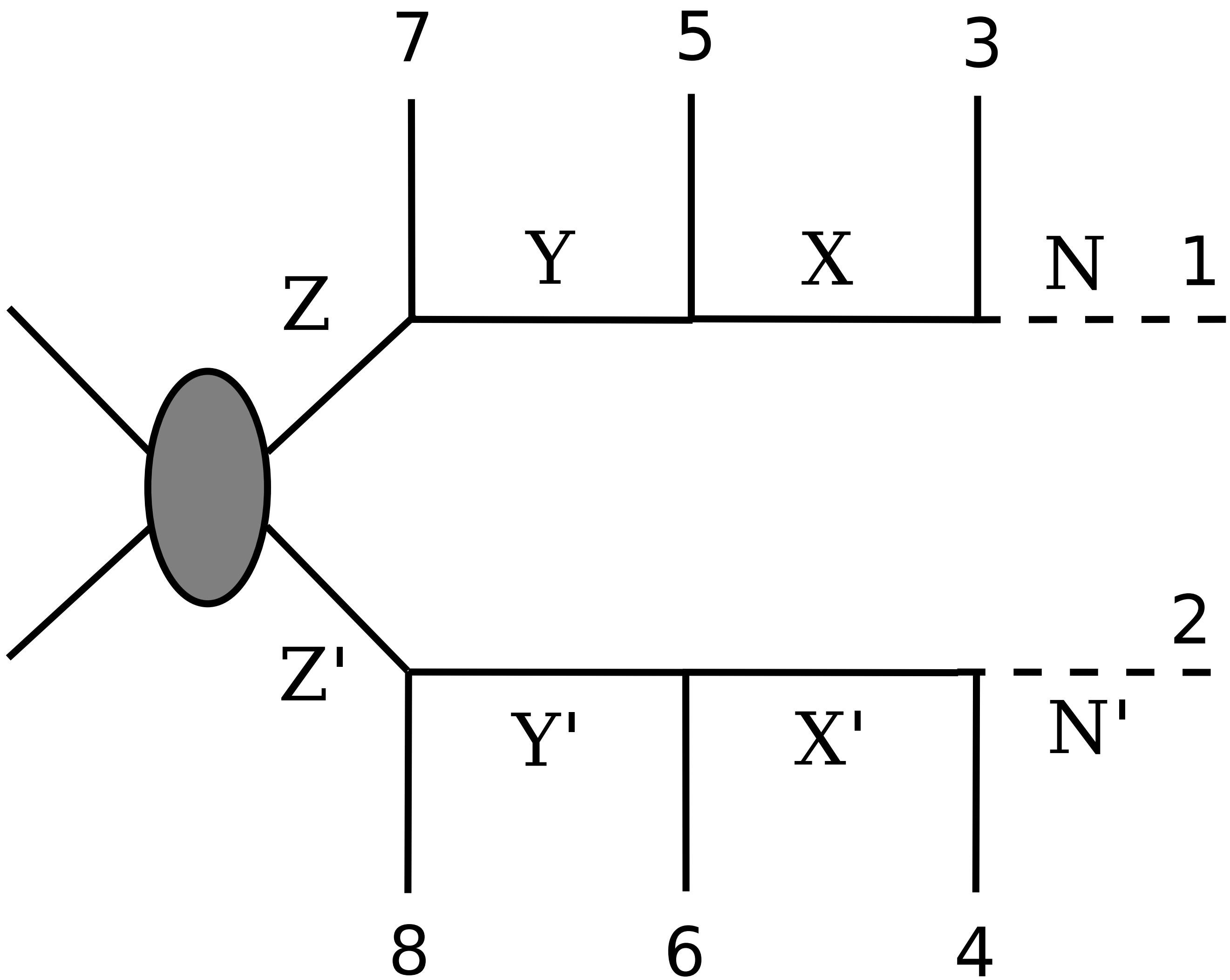
This is equivalent to asking: Is the 2-particle likelihood, in the narrow-width approximation zero or non-zero?

$$L_2 = P_1(\{p_i\}_1|\{M_j\})P_2(\{p_i\}_2|\{M_j\})$$

Naively this gives 4 quadratic equations. However one can use instead three quadratics by relating momenta $p_1^2 = p_2^2$.

Another nice way to think of this is doing OSET's *backwards*.

Event Pair Topology



Constraint Equations

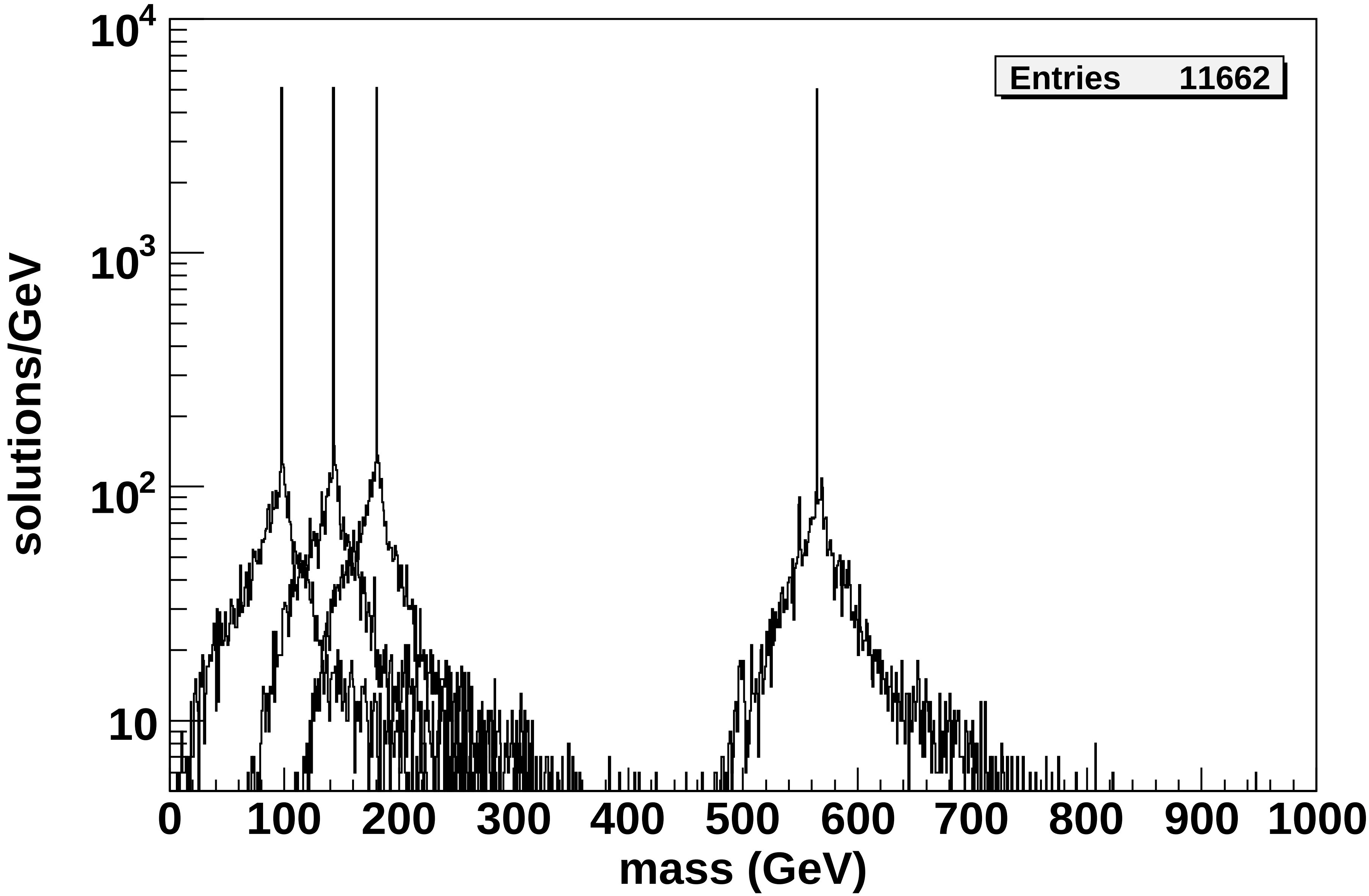
$$
\begin{array}{rcrcl}
(M_Z^2 & =) & (p_1+p_3+p_5+p_7)^2 & = & (p_2+p_4+p_6+p_8)^2, \\
(M_Y^2 & =) & (p_1+p_3+p_5)^2 & = & (p_2+p_4+p_6)^2, \\
(M_X^2 & =) & (p_1+p_3)^2 & = & (p_2+p_4)^2, \\
(M_N^2 & =) & p_1^2 & = & p_2^2.
\end{array}
\tag{2}
$$

$$p_1^x + p_2^x = p_{miss}^x, \quad p_1^y + p_2^y = p_{miss}^y.$$

$$
\begin{array}{rcccl}
q_1^2 & = & q_2^2 & = & p_2^2, \\
(q_1+q_3)^2 & = & (q_2+q_4)^2 & = & (p_2+p_4)^2, \\
(q_1+q_3+q_5)^2 & = & (q_2+q_4+q_6)^2 & = & (p_2+p_4+p_6)^2, \\
(q_1+q_3+q_5+q_7)^2 & = & (q_2+q_4+q_6+q_8)^2 & = & (p_2+p_4+p_6+p_8)^2,
\end{array}
$$

$$q_1^x + q_2^x = q_{miss}^x, \qquad q_1^y + q_2^y = q_{miss}^y.$$

Ideal Masses (without combinatorics)



Application of Realism

- Combinatorics: There are 16 choices of where to assign the leptons/jets per event for 4μ or 4e, or 8 for 2μ2e. Combinatorics are fundamental and *must* be taken into account. There is no magic cut which gets rid of them. Combinatorics also *carry information about mass*.

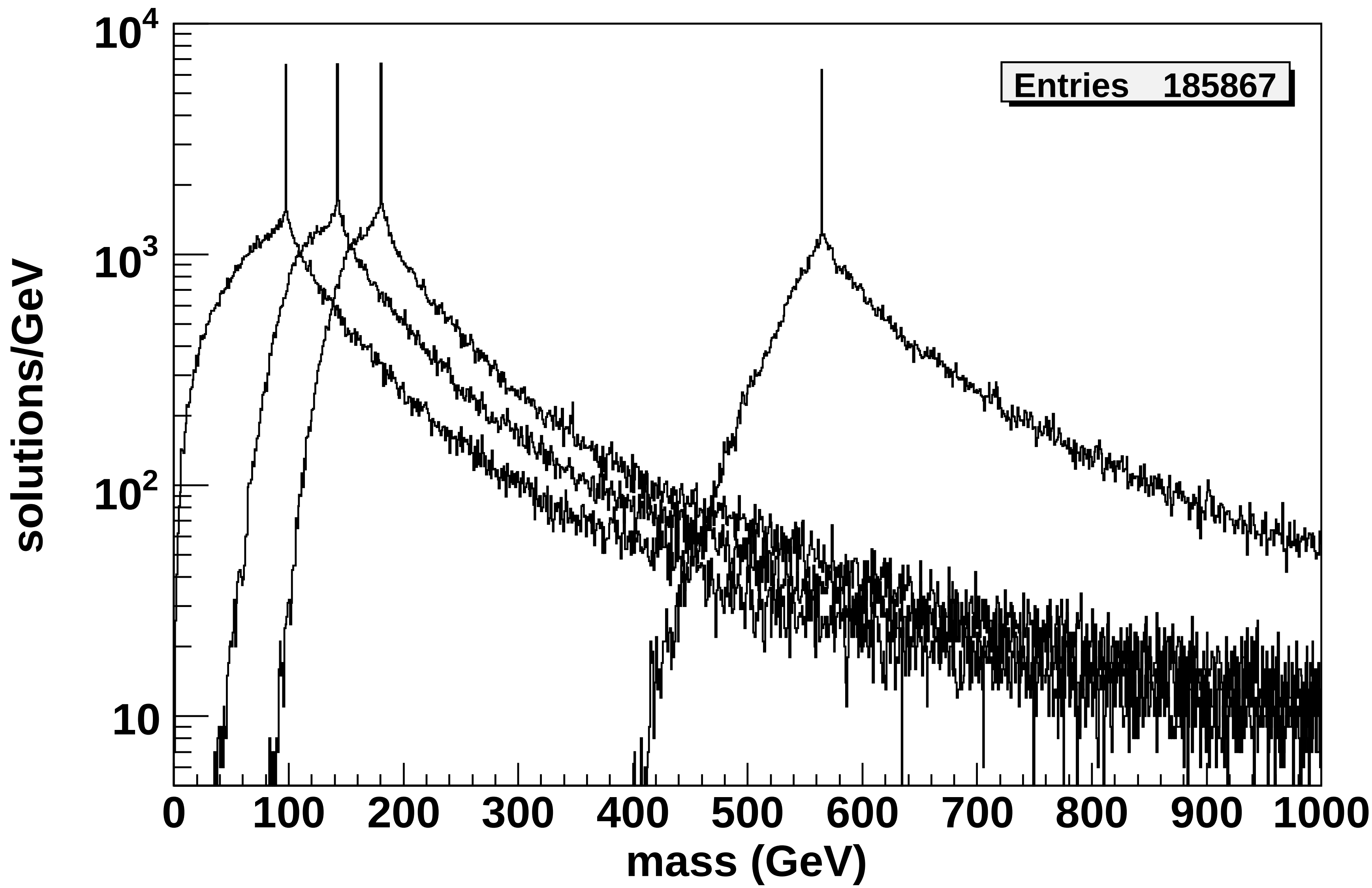
- Backgrounds: This signal has no real SM background. We include all SUSY backgrounds including $\tilde{\tau}$ decays and $\tilde{\chi}_2^0$ not from squark decay, and $\tilde{g}$ events (which have extra hard jets).

- Finite widths: $\Gamma_{\tilde{q}} = 5$ GeV, $\Gamma_{\tilde{\chi}_2^0} = 20$ MeV, $\Gamma_{\tilde{\ell}_R} = 200$ MeV.

- Mass splitting: Different flavor squarks have different masses by 6 GeV. Therefore, our squark mass result is an average of these signals.

Note that these techniques work with *very* few events (e.g. ten).

Ideal Masses (with combinatorics)



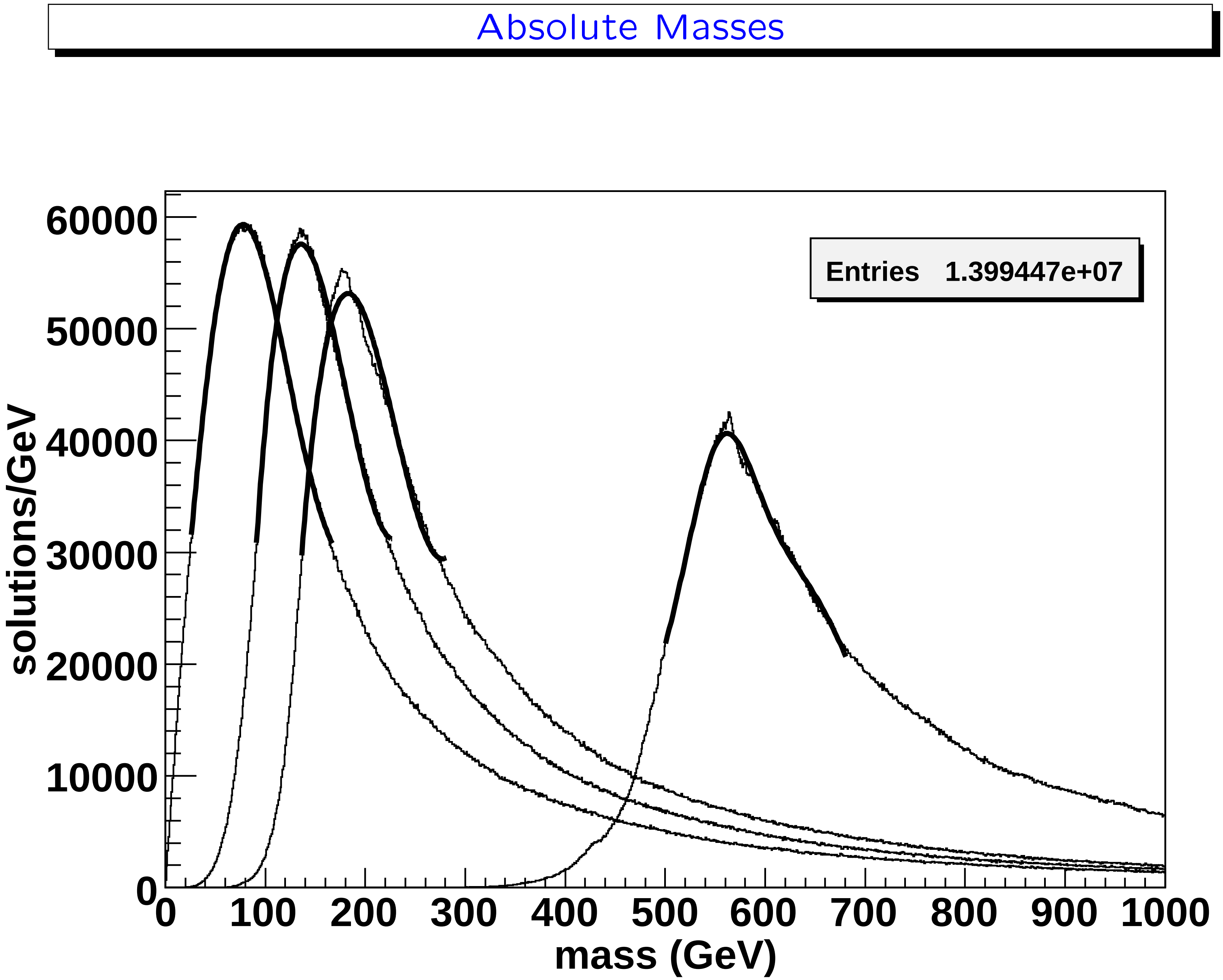
Application of Realism

We simulate all events with ATLFAST running in high-luminosity mode. We assume 300 fb^{-1} of luminosity. We require

- 4 isolated ($\Delta R < 0.4$) leptons with $p_T > 10$ GeV, $|\eta| < 2.5$. (flavors, charges chosen to match our $\widetilde{\chi}_2^0 \to \widetilde{\ell} \to \widetilde{\chi}_1^0$ topology.

- no b-jets and ≥ 2 jets with $p_T > 100$ GeV, $|\eta| < 2.5$. The highest p_T jets are taken to be particles 7,8 (extra jets from parton shower/reconstruction are present).

- Missing $p_T > 50$ GeV.



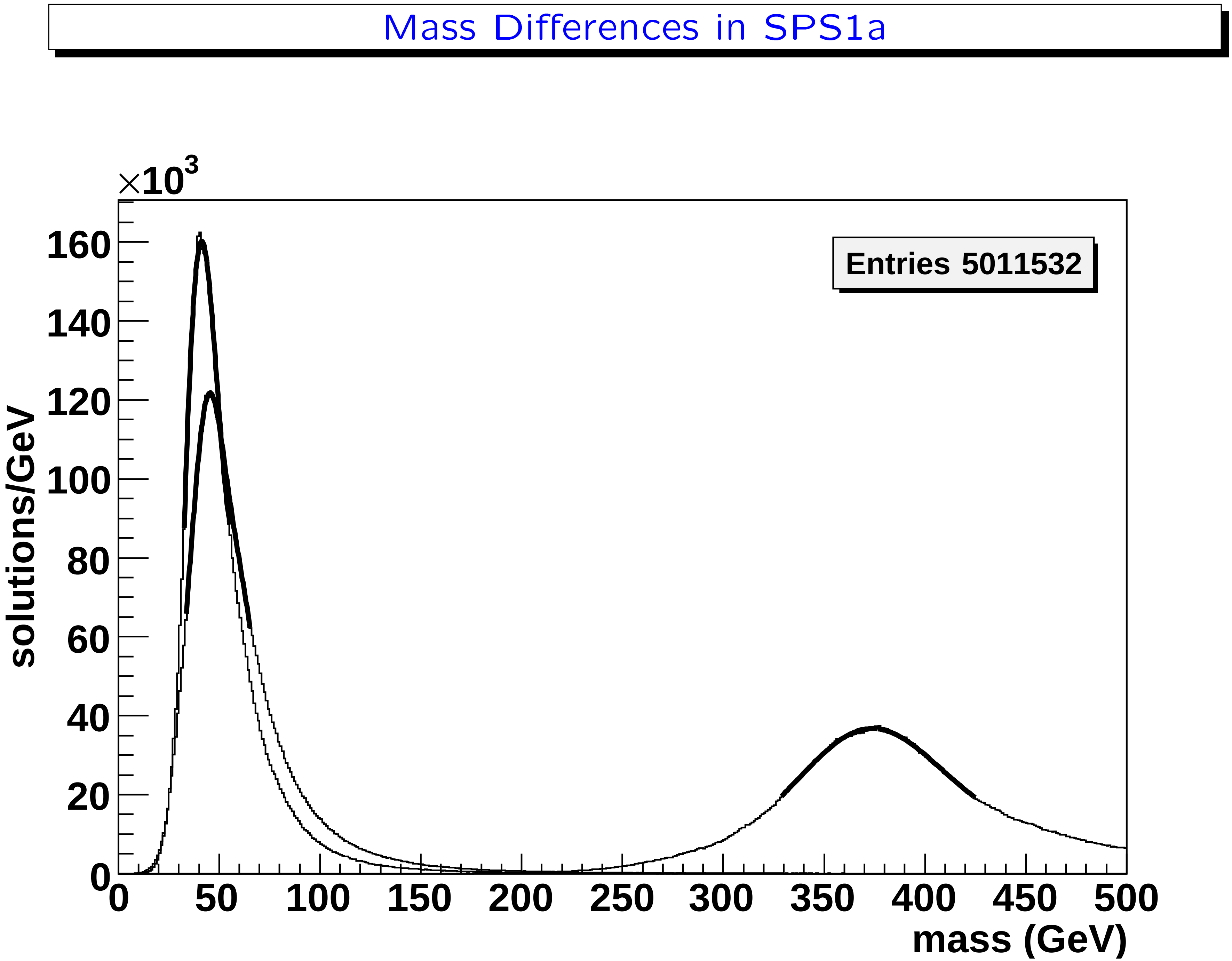
Absolute Masses
Entries 1.399447e+07
solutions/GeV
mass (GeV)
0
10000
20000
30000
40000
50000
60000
0
100
200
300
400
500
600
700
800
900
1000

Extra Cuts

We add new cuts to improve S/B and decrease bias

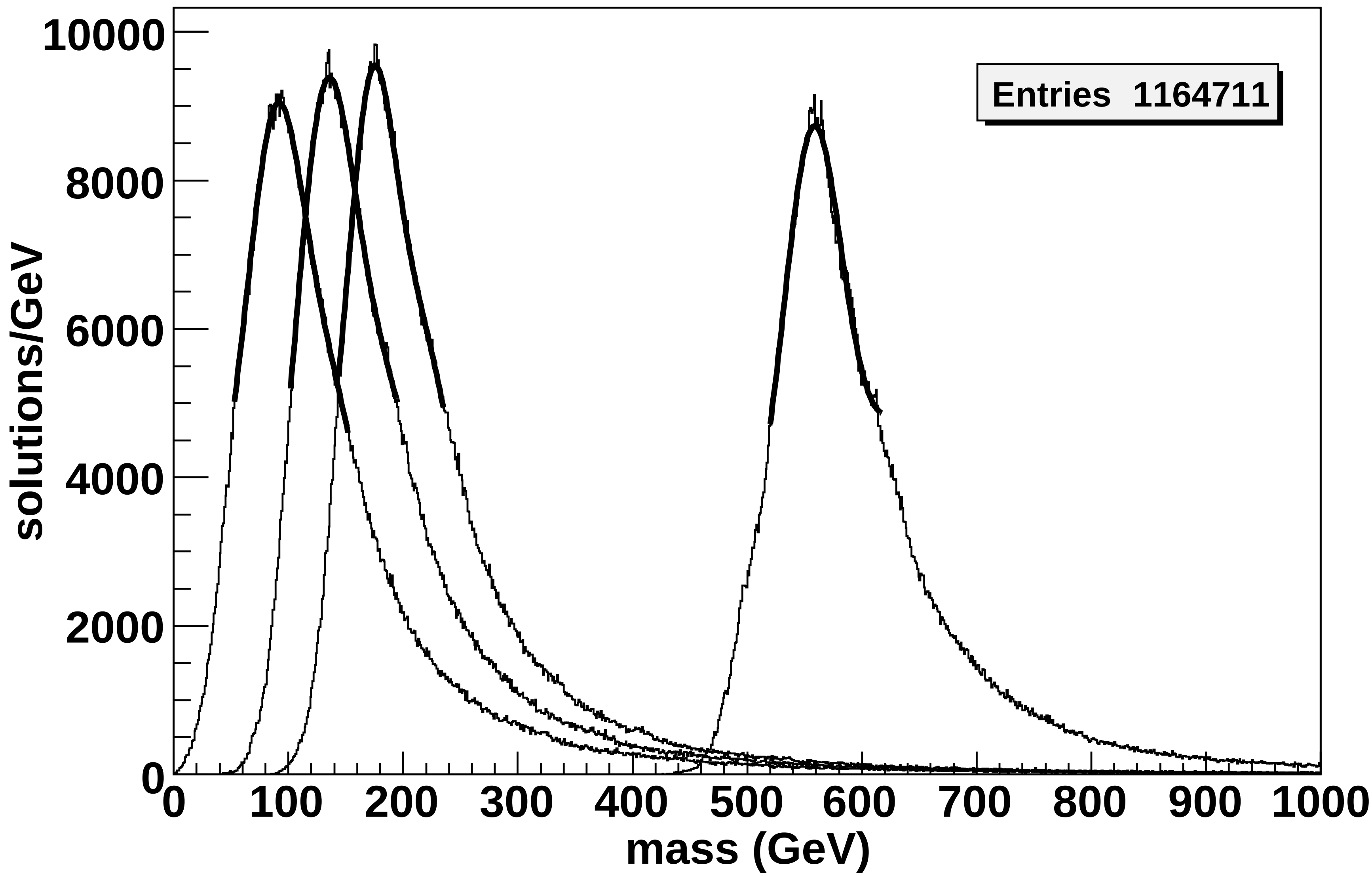
- We require that each combination c in each event i have solutions with some combination in 75% of the other events. $N_{pair}(c,i) < 0.75 N_{events}$
- We weight the final histogram by $1/N$ where N is the number of solutions in a given pair.
- We cut on the mass differences (window defined by 0.6 of peak height – e.g. Full Width at 0.6 Max)

There are many other interesting manipulations one can do, that are quite different from cutting on physical observables.



Mass Differences in SPS1a
×10^3
Entries 5011532
solutions/GeV
0
20
40
60
80
100
120
140
160
0
50
100
150
200
250
300
350
400
450
500
mass (GeV)

Absolute Masses SPS1a (cuts on ΔM)



Results

We fit peaks using a gaussian+quadratic polynomial, and use the maximum as our mass estimator. This is a biased estimator, but can be used to estimate our statistical error by repeating the measurement. Using 10 independent sets of Monte Carlo, for the SPS1a point with masses {91.7,135.9, 175.7 558.0}

$$\begin{aligned} m_N &= 94.1 \pm 2.8\text{GeV}, \\ m_X &= 138.8 \pm 2.8\text{GeV}, \\ m_Y &= 179.0 \pm 3.0\text{GeV}, \\ m_Z &= 561.5 \pm 4.1\text{GeV}. \end{aligned} \tag{3}$$

There are 539 signal + 195 background events in this sample after all cuts.

Precision is degraded by our "bias reduction" procedure. This is great for getting the mass within 5% very quickly (without scanning in masses), but final errors using these techniques is about a factor 2 better.

How to apply this to other processes

We used 300 fb^{-1} to make contact with other SPS1a studies, but this works with as few as 1 event.

Unknown R	Missing M	Events	Quadratics	Solutions
4	2	∞ (*)	2	0
5	2	3	5	32
6	2	4	6	64
7	2	4	7	128
8	2	5	8	256
7	3	∞ (*)	3	0
8	3	5	13	8192
9	3	5	14	16384

The assumption that two resonances in an event have the same mass and known resonances (Such as M_W, M_t) reduce the number of quadratics/events needed. Each event contributes $3M-2$ unknown mising momenta and generates M quadratics in $R+1$ unknown masses. One quadratic is always redundant.

* See Cheng, Gunion, Han, Marandella, McElrath JHEP 0712:076,2007

Summary

We *really can* make plots of mass!

Breit-Wigners appear in plots of mass, and the appearance of a Breit-Wigner is *real proof* of a new particle. Edges/slopes are far less convincing that one has discovered a new particle and not a detector effect (or a misinterpretation of a resonance as an edge!)

These techniques can be thought of as answering: Is the N-particle narrow-width likelihood L_N zero or non-zero?

These techniques require $\geq$ 4 resonances for 2 missing particles, or $\geq$ 7 resonances for 3 missing particles.

These techniques use *all* available data, (including missing p_T) and automatically take into account the fact that there are multiple solutions and combinatorics.

If the signal nature presents us is compatible with these requirements, this is really the the best, unambiguous variable to use.