QCD Reummation for Heavy Quarkonium Production in High Energy Collisions

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based on work with J. -W. Qiu

Success of NRQCD

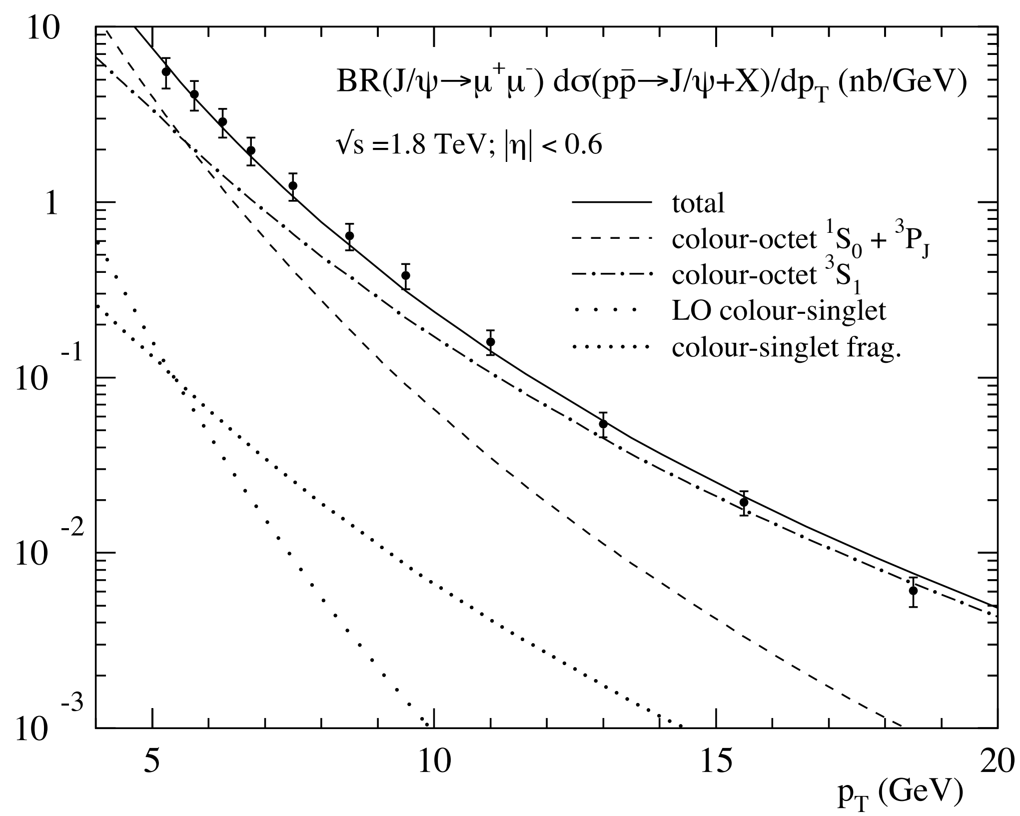
NRQCD approach for quarkonium production

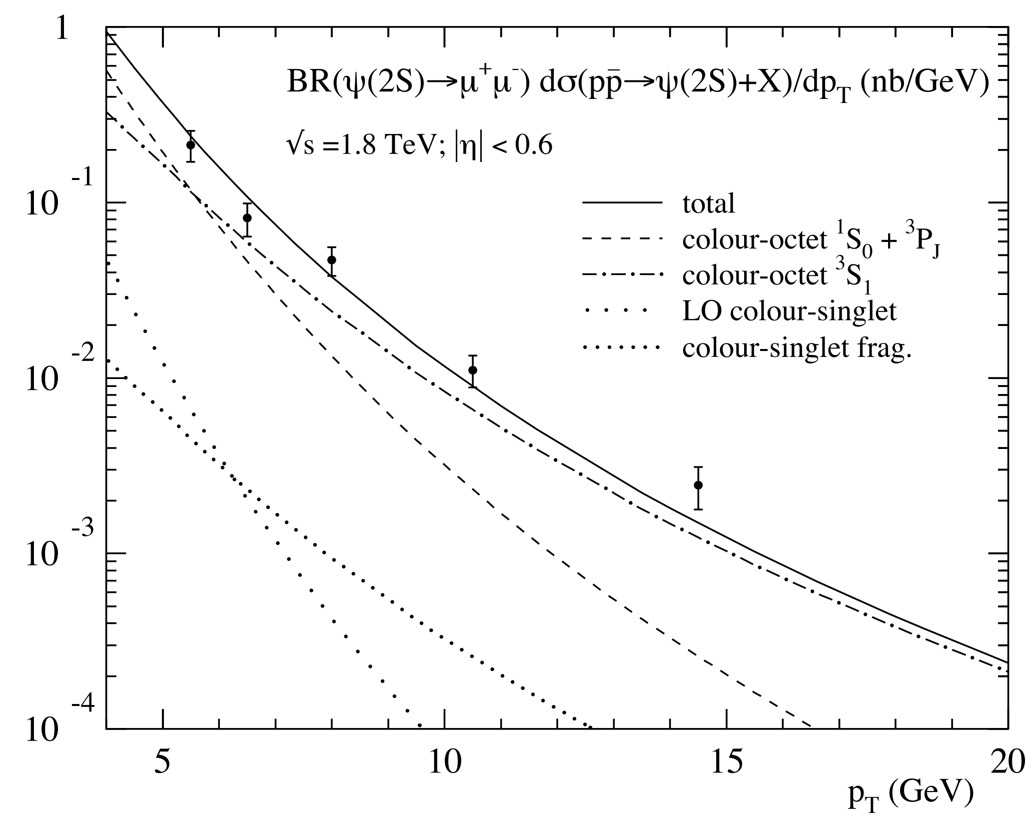
$$\sigma(pp \to H + X) = \sum_{i,j,n} \int dx_1 dx_2 \phi_{i/p}(x_1)\phi_{j/p}(x_2)\hat{\sigma}\left[ij \to (Q\bar{Q})_n\right]\langle O_n^H \rangle$$ Braaten, Bodwin, Lepage 1995

$\hat{\sigma}\left[ij \to (Q\bar{Q})_n\right]$: production of $Q\bar{Q}$ state with quantum number n, calculable in pQCD as a expansion of α_s

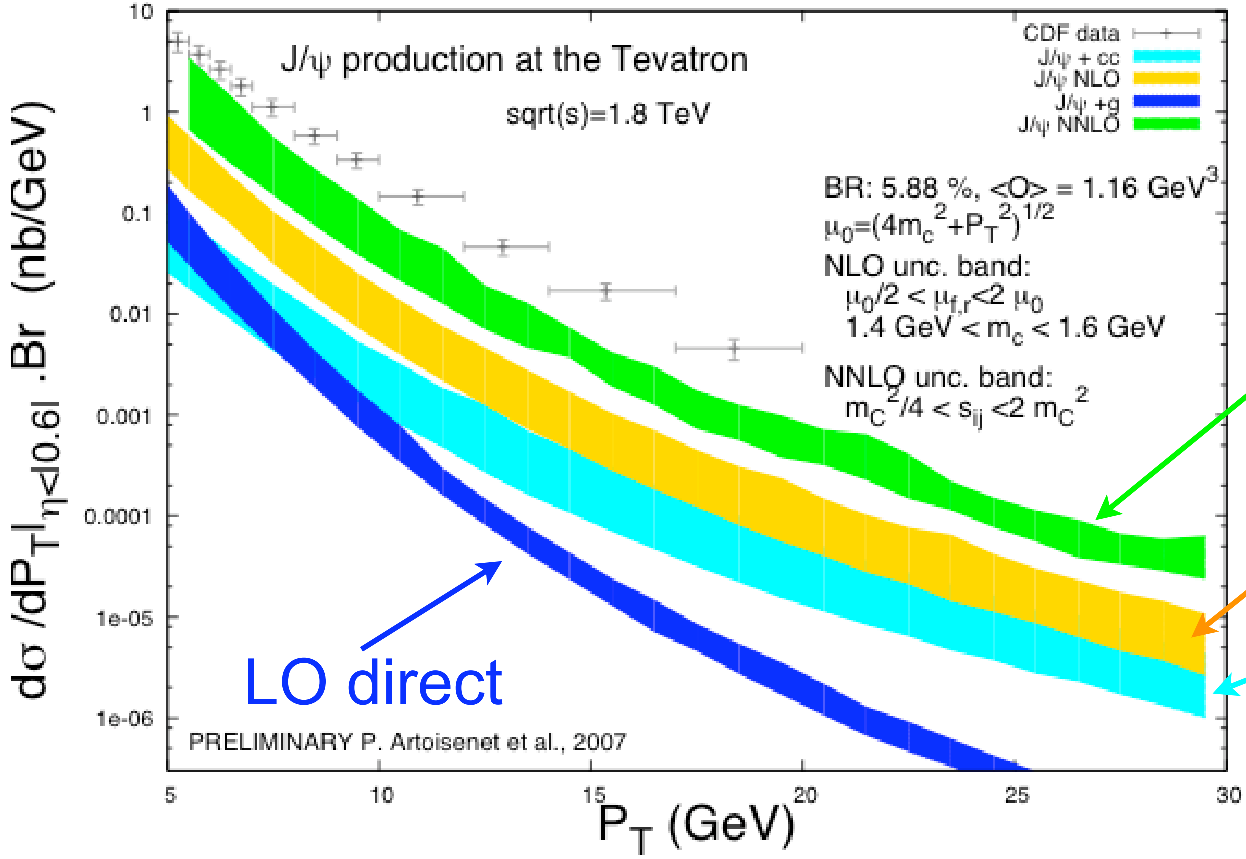
$\langle O_n^H \rangle$: can be expanded in powers of v^2

Comparison with Tevatron data based on LO formula





NLO contributions



Color-singlet contribution for J/ψ and Upsilon production at Tevatron

NNLO

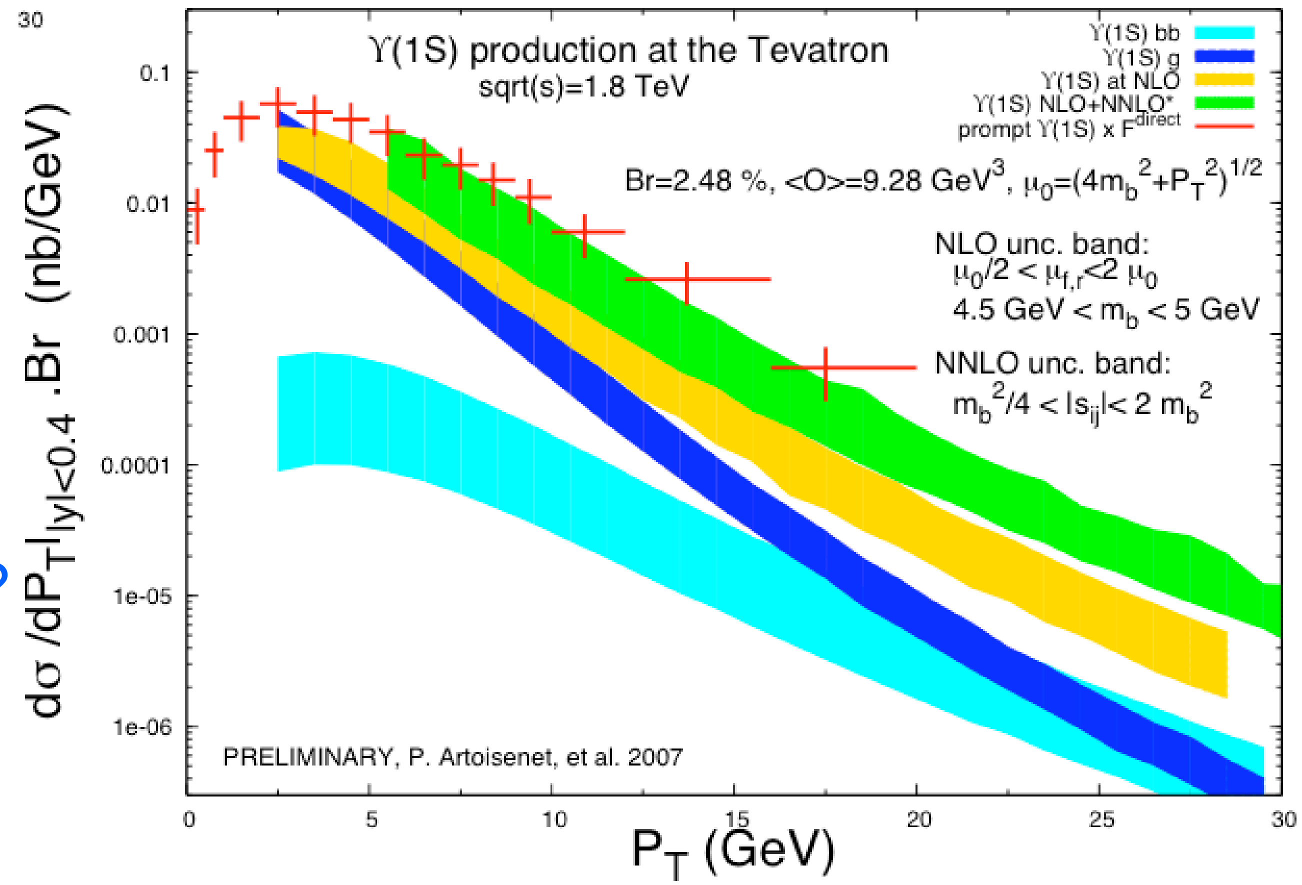
P. Artoisenet, F. Maltoni, et.al. 2007

NLO

LO associate

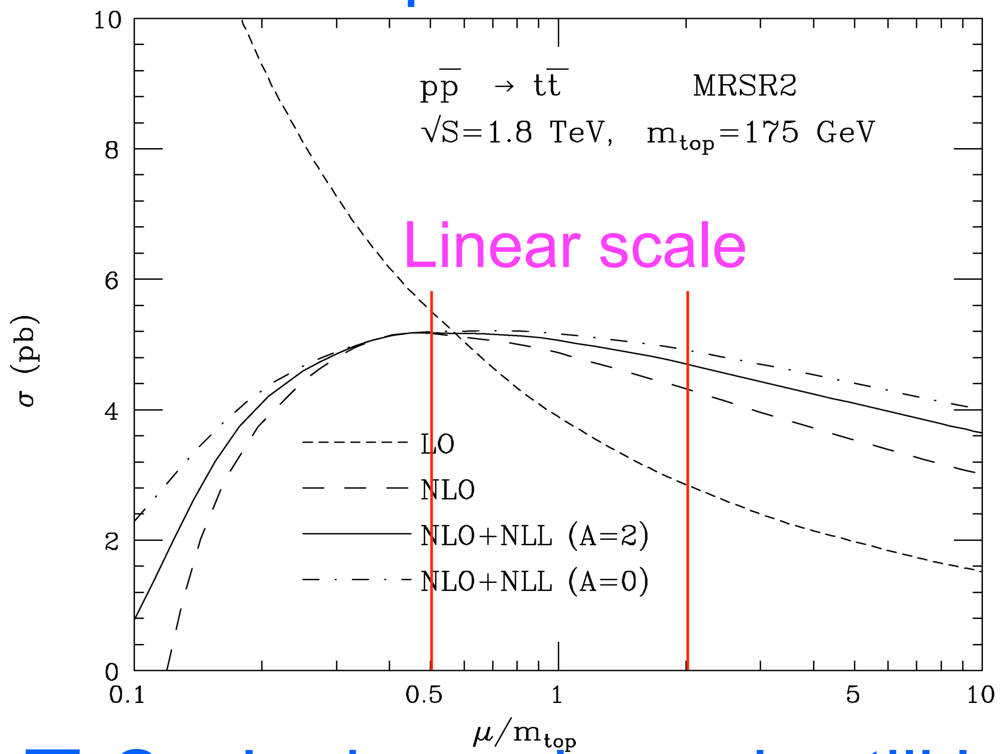
Large uncertainty band
⇒strong scale dependence

Large NLO, NNLO contribution
⇒ how perturbative series converge?



Scale dependence of the cross section

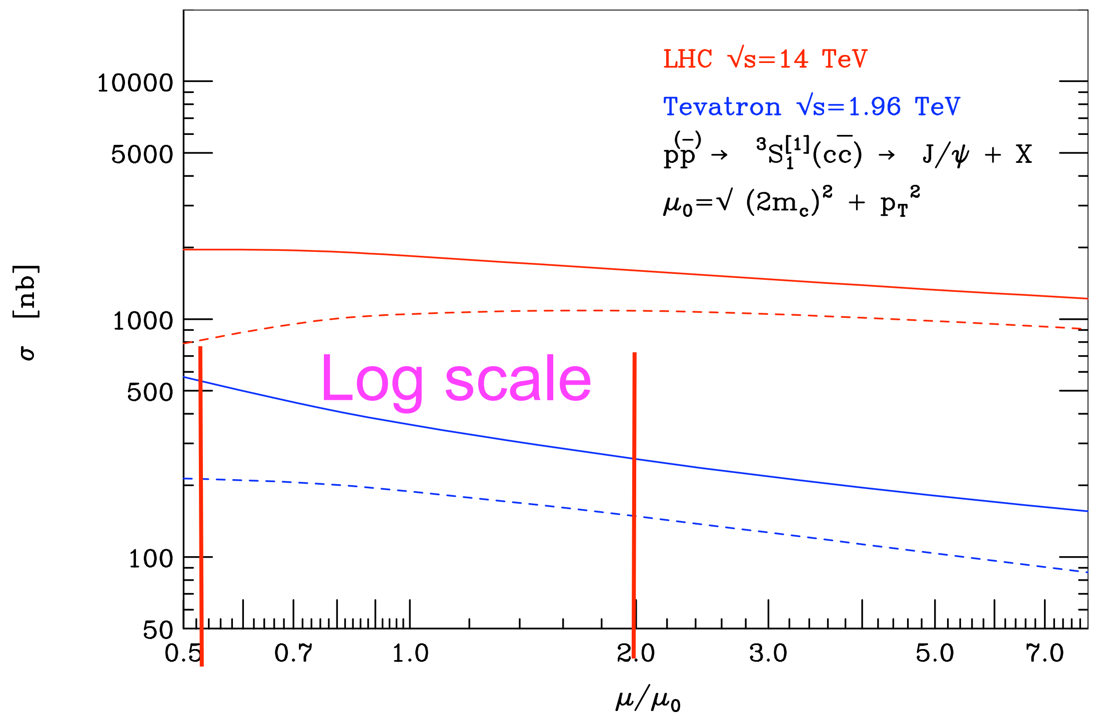
- Scale dependence of the ttbar cross section at NLO



With NLO correction included, scale-dependence is strongly reduced

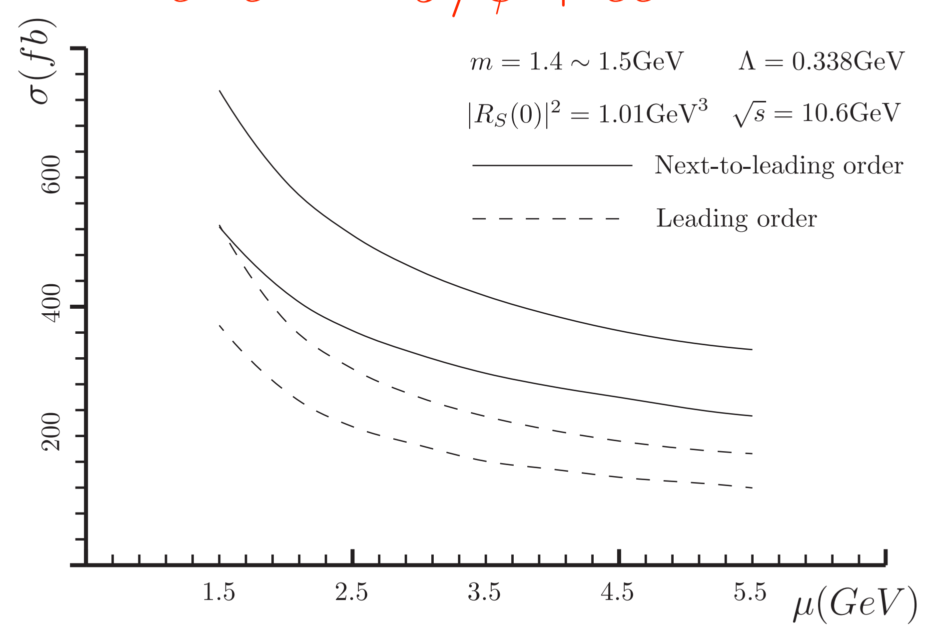
Bonciani, Catani, Mangano, Nason, NPB529 (1998) 424

- Scale dependence is still large for J/ψ at NLO: large NLO corrections



Campbell, Maltoni, Tramontano, PRL98(2007) 252002

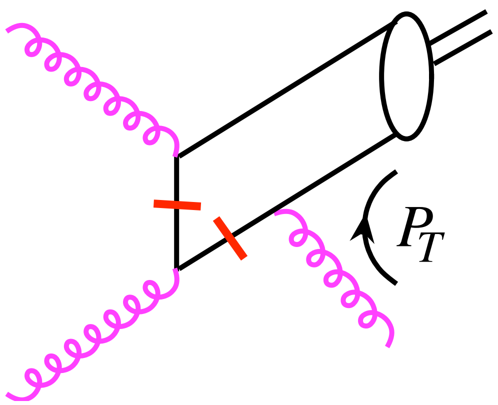
$$e^+e^- \to J/\psi + c\bar{c}$$



Zhang and Chao PRL98, 092003(2007)

Why NLO contribution is LARGE?

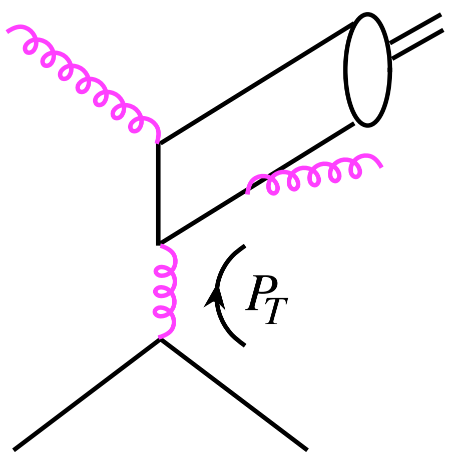
- LO



$$\alpha_s^3 \frac{(2m)^4}{P_T^8}$$

- Scale dependence from $\phi(x,\mu)^2 \alpha_s^3(\mu)$
- P_T dependence $\frac{1}{P_T^8}$

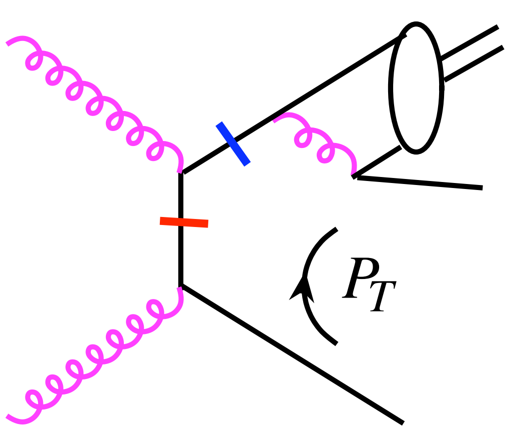
- NLO: new channel



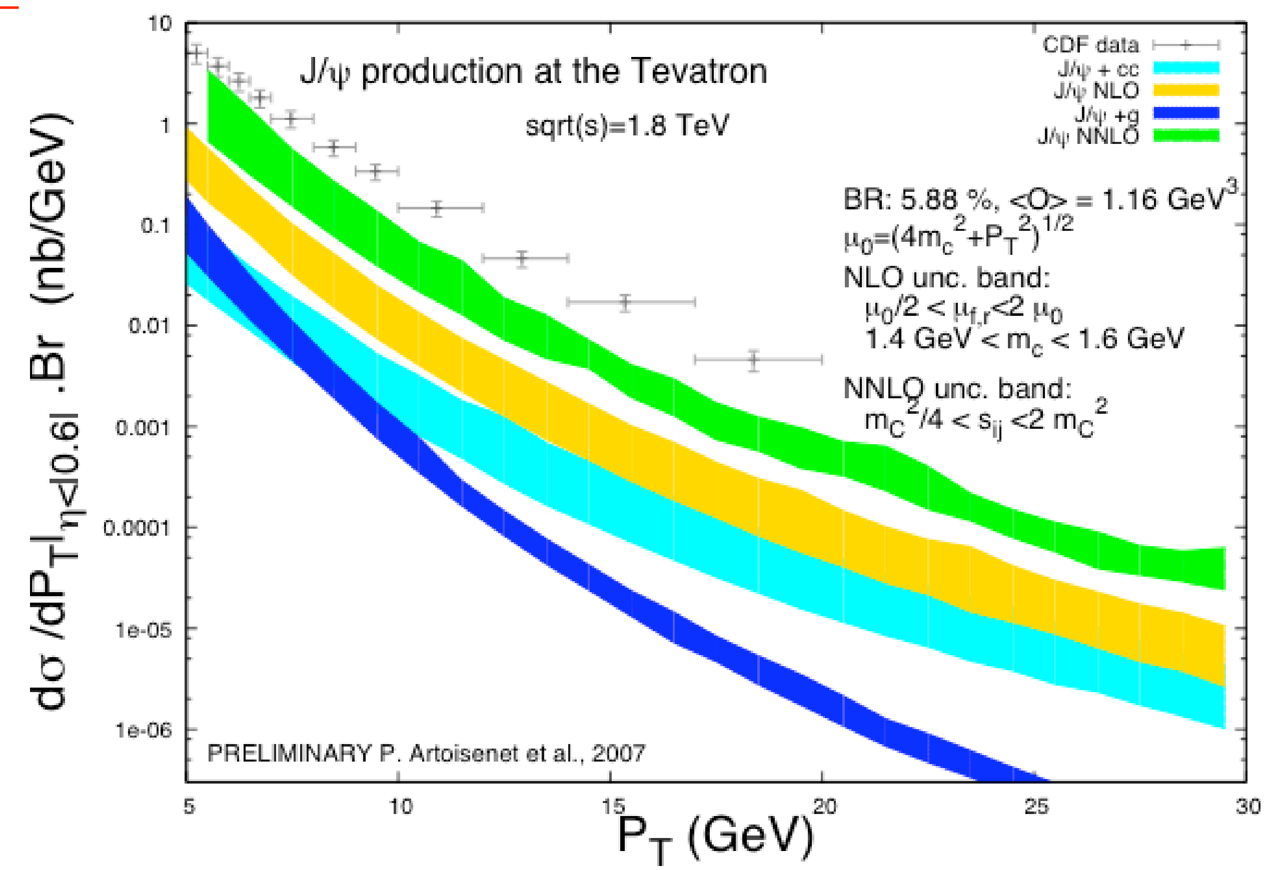
$$\alpha_s^4 \frac{(2m)^2}{P_T^6}$$

NLO: high power $\alpha_s(\mu)$, low power in P_T

- NLO to existing LO channels

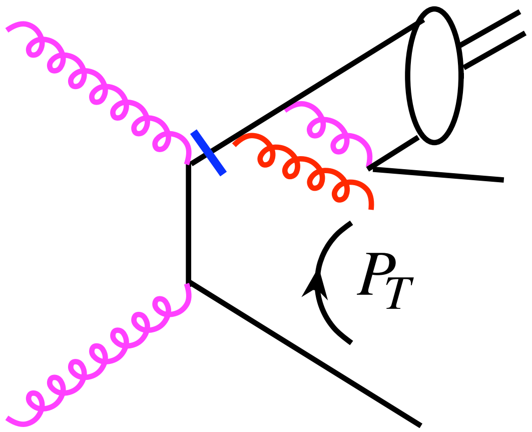


$$\alpha_s^4 \frac{1}{P_T^4}$$



Large logarithmic contributions

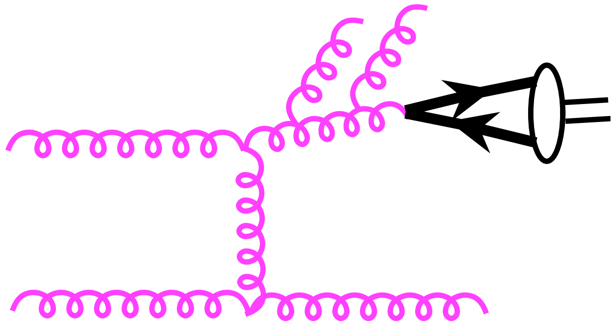
- NNLO



$$\alpha_s^4 \frac{1}{P_T^4} \cdot \left(\alpha_s \ln \left[\frac{P_T^2}{m^2} \right] \right)$$

- To have a stable perturbative expansion, one need resum all the large logarithms: resummation

- Same large log contribution for color-octet channels



New factorized formula with QCD resummation

- **Fragmentation contributions** E. Braaten, et.al., 1993

$$\sigma^F(pp \to H+X) = \sum_{i,j,k} \int dx_1 dx_2 dz \phi_{i/p}(x_1)\phi_{j/p}(x_2)\hat{\sigma}\,[ij \to k] D_{k\to H}(z)$$

$D_{k\to H}(z)$ resums all the logarithms.
This is the dominant contribution when $P_T^2 >> m^2$

- Q: What is the relation between fragmentation contribution and fixed order results in NRQCD?

$P_T^2 \sim m^2 : \quad \sigma \approx \sigma^{Pert}$ calculated by fixed order NRQCD. Logarithms are not important

$P_T^2 \gg m^2 : \quad \sigma \approx \sigma^F$ Logarithms dominate / resummed

- How to transform smoothly between these two regimes?
- How to avoid double counting beyond LO?

- **We propose a new factorized formula:**

$\sigma = \sigma^{Dir} + \sigma^F$ ← resum all the fragmentation logs

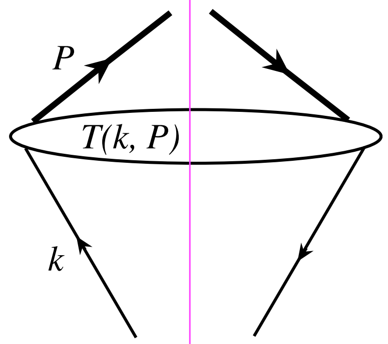
$\sigma^{Dir} = \sigma^{Pert} - \sigma^{Asym}$ ← No logs

separation between Direct and Fragmentation contribution depends on the definition of fragmentation function $D(z, \mu^2)$

Fragmentation function $D_{q\to J/\psi}(z_f,\mu^2)$

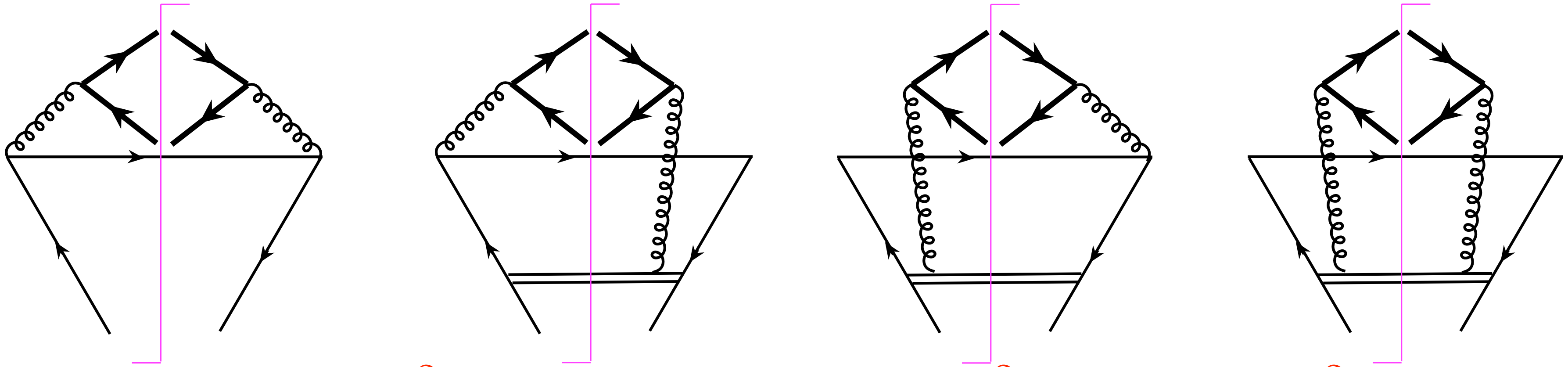
- Operator definition for $D_{q\to J/\psi}(z_f,\mu^2)$

$$D_{k\to H}(z_f,\mu^2) =$$



$$= \int_{k^2\le\mu^2} \frac{d^4k}{(2\pi)^4}\frac{z_f^2}{4k^+}\delta(z_f - \frac{P^+}{k^+})\mathrm{Tr}\left[\gamma^+ T(k,P)\right]$$

- Calculation of leading order fragmentation function: $D^{(0)}{}_{q\to J/\psi}(z_f,\mu^2)$



$$D^{(0)}_{q\to J/\psi}(z_f,\mu) = \frac{\alpha_s^2}{36m^3}\langle O_8(^3S_1)\rangle \cdot \left[\frac{(z_f-1)^2+1}{z_f}\ln\left(\frac{z_f\mu^2}{4m^2}\right) - z_f\left(1-\frac{4m^2}{z_f\mu^2}\right)\right]$$

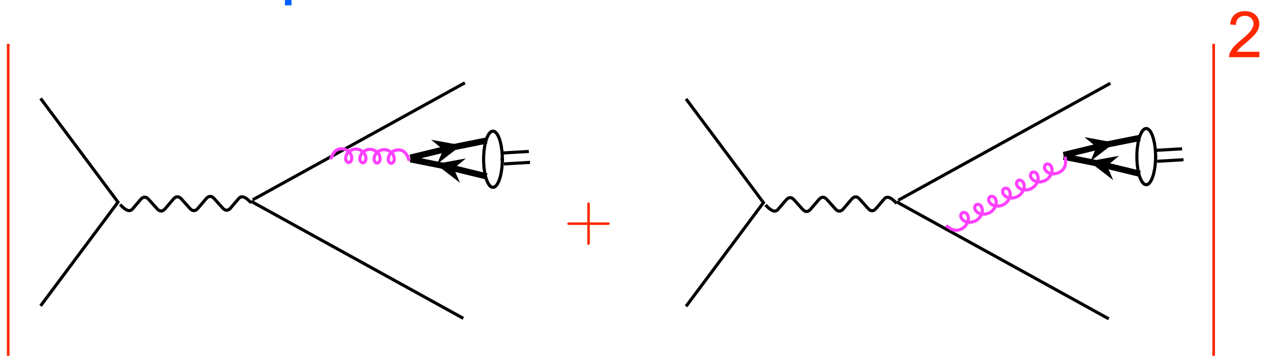
- Evolution equation of $D_{q\to\psi}(z_f,\mu^2)$: inhomogeneous term

$$\mu^2\frac{d}{d\mu^2}D_{q\to J/\psi}(z_f,\mu) = \gamma_{q\to J/\psi}(z_f,\mu) + \frac{\alpha_s}{2\pi}\int_{z_f}^1\frac{d\xi}{\xi}P_{q\to q}\left(\frac{z_f}{\xi}\right)D_{q\to J/\psi}(\xi,\mu) + \cdots$$

$$\gamma_{q\to J/\psi}(z_f,\mu) = \frac{\alpha_s^2}{36m^3}\langle O_8(^3S_1)\rangle\left[\frac{(z_f-1)^2+1}{z_f} - \frac{4m^2}{\mu^2}\right]\theta\left(\mu^2 - \frac{4m^2}{z_f}\right)$$

Case study: $e^+e^-\to J/\psi+q\bar{q}$

☐ NRQCD perturbative results



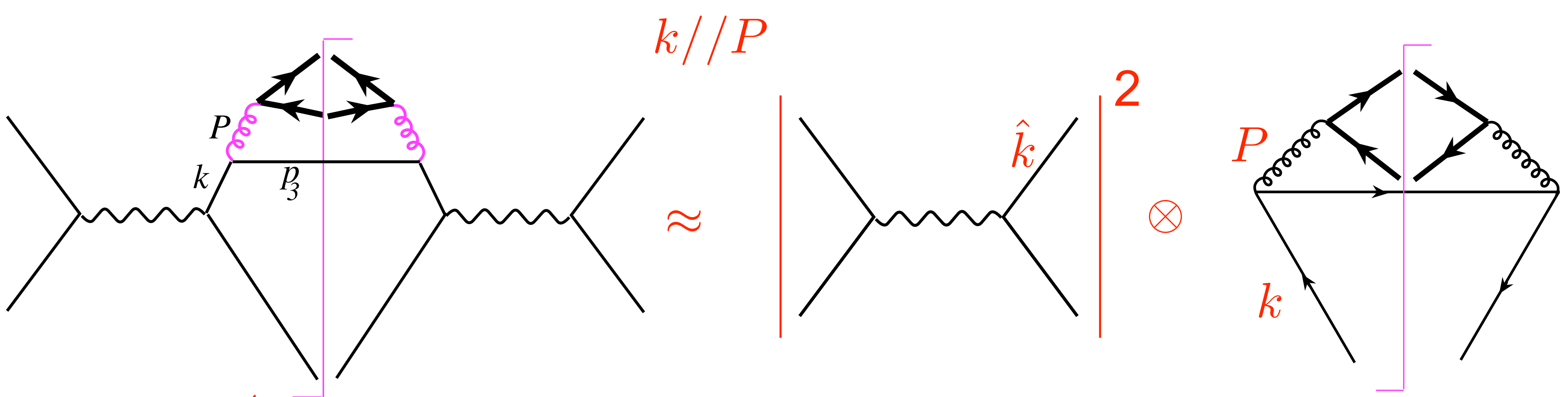
$$z=\frac{2E_{J/\psi}}{\sqrt{s}} \qquad \xi=\frac{4m^2}{s}$$

$$z_L=\sqrt{z^2-4\xi}$$

$$\ln\left(\frac{E^2_{J/\psi}}{zm^2}\right)$$

$$\frac{d\sigma^{Pert}}{dE_{J/\psi}}=\sigma_0\cdot\frac{2}{\sqrt{s}}\frac{\alpha_s^2}{18}\frac{\langle O_8(^3S_1)\rangle}{m^3}\left[\left(\frac{(z-1)^2+1}{z}+2\xi\frac{2-z}{z}+\xi^2\frac{2}{z}\right)\ln\frac{z+z_L}{z-z_L}-2z_L\right]$$

☐ How to identify the logarithms before the full calculations $\longrightarrow$ σ^{Asym}



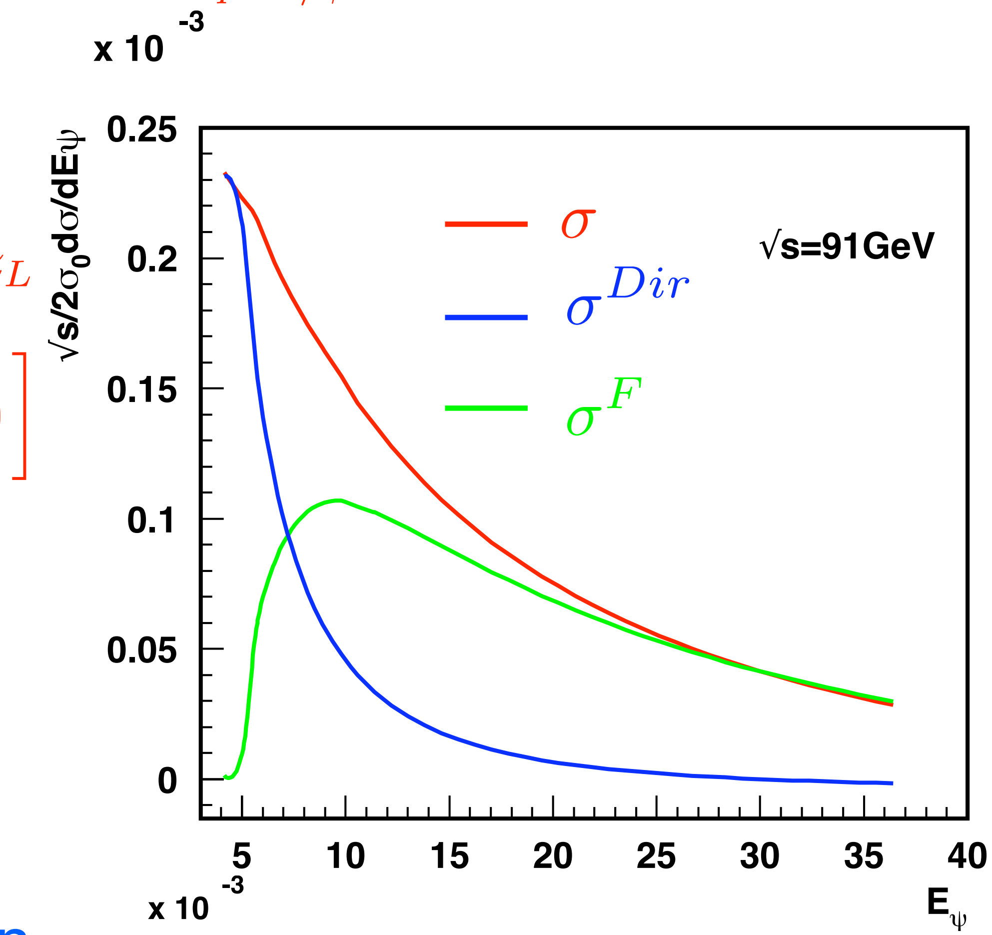
$$\frac{d\sigma^{Asym}}{dE_{J/\psi}}\approx\sigma_0\cdot D^{(0)}_{q\to J/\psi}(z_f,\mu^2,4m^2)\frac{dz_f}{dE_{J/\psi}} \qquad z_f=\frac{P^+}{k^+}=\frac{1}{2}\left[z+z_L\right]$$

Smooth transition

- **Direct contribution** $\sigma^{Dir} = \sigma^{Pert} - \sigma^{Asym} = \sigma^{Pert} - 2\sigma_0 \cdot D^{(0)}_{q\to J/\psi}(z,\mu^2)$

$$\frac{d\sigma^{Dir}}{dE_{J/\psi}} = \sigma_0 \cdot \frac{2}{\sqrt{s}} \frac{\alpha_s^2}{18} \frac{\langle O_8(^3S_1)\rangle}{m^3} \times \left[\left(\frac{(z-1)^2+1}{z} + 2\xi\frac{2-z}{z} + \xi^2\frac{2}{z}\right)\ln\frac{z+z_L}{z-z_L} - 2z_L - \frac{z_f}{z_L}\left(\frac{(z_f-1)^2+1}{z_f}\ln\left(\frac{z_f\mu^2}{4m^2}\right) - z_f\left(1-\frac{4m^2}{z_f\mu^2}\right)\right)\right]$$

$$\mu = 2E_{J/\psi}$$



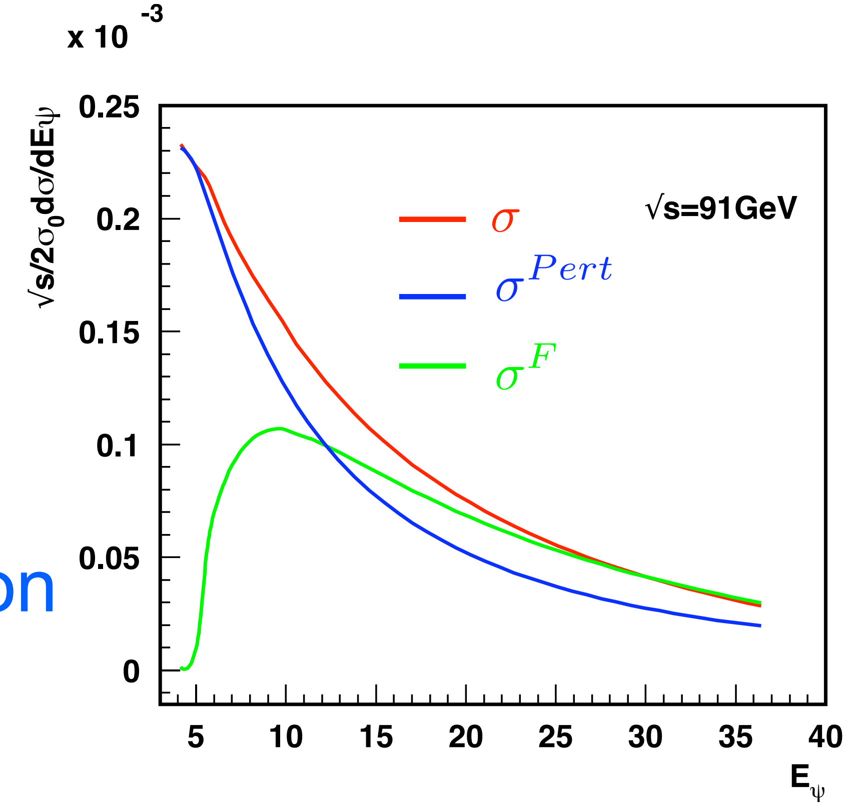
- **Full cross section**

$$\sigma = \sigma^{Dir} + \sigma^F$$

with evolved fragmentation function
⇒log resummed

- when $E_{J/\psi}$~m $\frac{d\sigma}{dE_{J/\psi}} \sim \frac{d\sigma^{Dir}}{dE_{J/\psi}}$
- when $E_{J/\psi}$>>m $\frac{d\sigma}{dE_{J/\psi}} \sim \frac{d\sigma^{F}}{dE_{J/\psi}}$

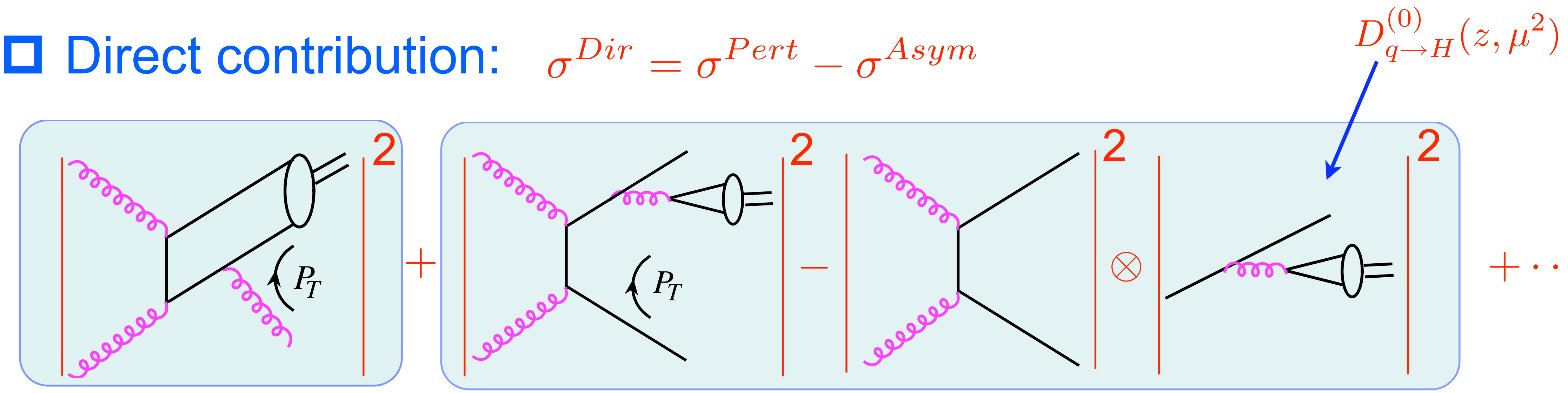
- **Compare to lowest order NRQCD calculation**



Hadronic collisions - in progress

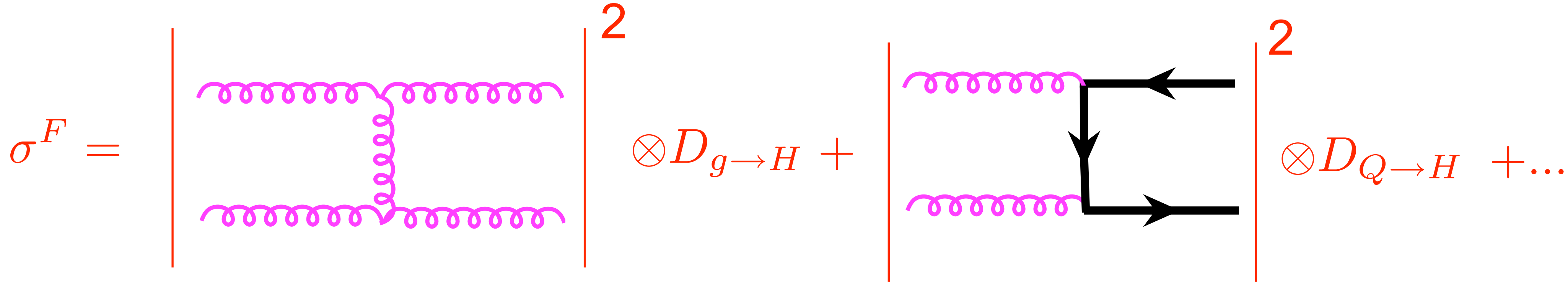
$$\sigma = \sigma^{Dir} + \sigma^{F}$$

- Direct contribution: $\sigma^{Dir} = \sigma^{Pert} - \sigma^{Asym}$



LO NLO

- Fragmentation contribution:



Stay tuned

Summary

- We proposed a QCD resummed factorization formula for heavy quarkonium production

- We reorganized the perturbative series of NRQCD calculation

- New formula is reliable for a wide range of collision energy