Isajet-M and Unified analysis of SUSY GUTs

Analyzing flavor & neutrino sectors with full running

Ali Soleimani

University of Kansas

Pheno 2008 - April 29, 2008

Outline

1. Analyzing Field Theories at Multiple Scales
2. Isajet-M Architecture
3. Results
4. Future work

Isajet-M

RGE analysis of SUSY GUTs

We want to analyze SUSY GUTs in conjunction with low-scale data.

- We know masses, couplings, etc. at a low scale.
- We know coupling constants unify at some high scale.
- Generally, we have some ansatz for Yukawas, SSB parameters, etc. at the high scale.
- They are connected by the RGEs.

So…we have a set of ODEs with mixed boundary conditions.

Approximations for the RGEs

RGEs are commonly solved in…

1. The *dominant third-family approximation.*
   - This assumes $Y_{ij} \approx \delta_{i3}\delta_{j3}$.
   - Captures largest effect in quark sector.
2. The *diagonal approximation*.
   - Assumes $Y_{ij} \approx \mathrm{diag}(y_{11},\ y_{22},\ y_{33})$
   - Better: the quark sector is approximately diagonal
   - But the lepton (neutrino) sector is not
3. Full three-family matrix couplings.
   - Treats all Yukawas in full generality

Difficulties with Matrix RGEs

- The number of parameters is greatly multiplied:
  - 3 Yukawa matrices eg $y_{u\,ij}$
  - 3 trilinear coupling matrices $h_{u\,ij}$
  - 5 mass-squared matrices $m^2_{\tilde{u}}$, $m^2_{\tilde{Q}}$, etc
  - 1 Yukawa matrix $y_{\nu\,ij}$
  - 1 trilinear coupling matrix $h_{\nu\,ij}$
  - 1 right-handed neutrino mass matrix $M_{\nu_r\,ij}$
  - 1 mass-squared matrix $m^2_{\tilde{\nu}\,ij}$
  - 1 effective coupling matrix κ_{ij}

  A total of $16 \times 9 = 144$ matrix parameters.
- Basis issues

Current Codes

Several codes exist to solve the SUSY β-functions:

- SuSpect
- SoftSUSY
- SPheno
- Isajet
- REAP
- Isajet-M

Only SoftSUSY handles full three-family couplings.
Only REAP handles the neutrino sector properly.
We want a code that does *all* of this.

What can we do with Isajet-M?

1. Flavor analysis
    - CKM constraints
    - FCNC constraints, b$\rightarrow s\gamma$
2. Neutrino sector
    - Neutrino mass splittings, mixings
    - Neutrino phases
3. Leptogenesis
4. Effects of neutrino RGEs on e.g. $\Omega_{\tilde{Z}_1}$ (cf. Barger, Marfatia, Mustafayev 2008)

Example Application

Bimaximal mixing

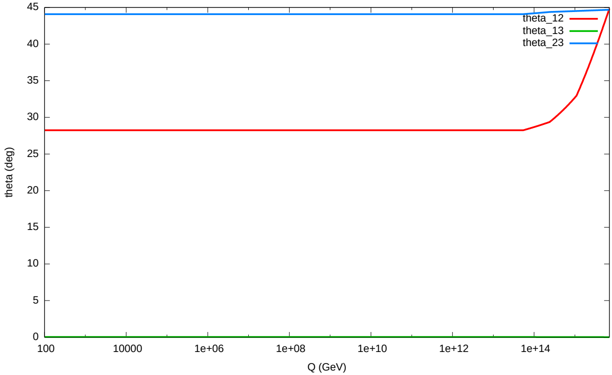


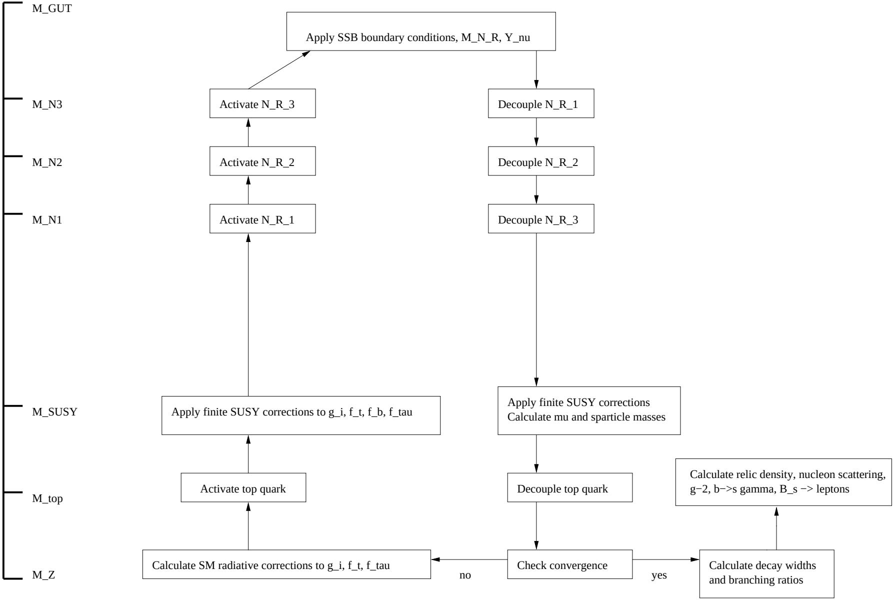
Figure: With Y_ν diagonal at the GUT scale, we see that θ_{12} runs significantly, while the other angles do not. Here we obtain bimaximal mixing at the GUT scale.

Isajet-M Layout

Overall scheme

1. Impose boundary conditions at M_Z
2. Run up to M_{GUT}
3. Impose GUT-scale boundary conditions
4. Run down to M_{SUSY}
5. Calculate $|\mu|$ and SUSY spectrum
6. Run back down to M_Z

Repeat until convergence.



M_GUT

M_N3

M_N2

M_N1

M_SUSY

M_top

M_Z

Apply SSB boundary conditions, M_N_R, Y_nu

Decouple N_R_1

Decouple N_R_2

Decouple N_R_3

Apply finite SUSY corrections
Calculate mu and sparticle masses

Decouple top quark

Check convergence

no

Calculate SM radiative corrections to g_i, f_t, f_tau

Activate top quark

Apply finite SUSY corrections to g_i, f_t, f_b, f_tau

Activate N_R_1

Activate N_R_2

Activate N_R_3

yes

Calculate decay widths
and branching ratios

Calculate relic density, nucleon scattering,
g−2, b−>s gamma, B_s −> leptons

Isajet-M Inputs

We take the following paramters as inputs:

1. SUSY model and parameters
    - mSUGRA: $m_0, m_{1/2}, A_0, \tan\beta, \mathrm{sgn}(\mu)$
2. SM particle masses and mixings
3. Higgs vev
4. Right-handed neutrino Majorana mass matrix M_N
5. Neutrino Yukawa matrix Y_ν

Isajet-M Outputs

We calculate the following quantities:

- Sparticle spectrum
- Higgs masses
- Decay widths and branching fractions
- Neutrino masses and mass splittings
- Neutrino mixing angles (and phases)
- $\Omega_{\tilde{Z}_1} h^2$, BF($b \to s\gamma$), $(g-2)_\mu$, $\sigma(\tilde{Z}_1 p)$, BF($B_s \to \ell\bar{\ell}$).

Low-scale boundary conditions

Boundary conditions at M_Z:

- Use measured masses, v, and V_{CKM} to find Y_e, Y_u, Y_d
- We run in $\overline{DR}$, so first:
  - Run masses to M_Z in 3-loop QCD plus 2-loop QED $\overline{MS}$.
  - Apply finite threshold corrections
  - Convert from $\overline{MS}$ into $\overline{DR}$
- Run from M_Z to m_{top}.
- Correct Y_{top} at m_{top}.

High-scale boundary conditions

Boundary conditions at M_{GUT}:

- M_{GUT}, determined by gauge unification
- Soft SUSY-breaking parameters:
  - Universal gaugino, scalar masses
  - Universal trilinear couplings
- M_{ν_r}, Y_ν

Decoupling

- Particles only contribute to RG evolution when they are kinematically accessible.
- At scales below their mass, they do not contribute.
- We change RGEs at mass thresholds:
  1. Sparticles
  2. Higgs, top quark
  3. Right-handed neutrinos
- Isajet includes finite threshold corrections for sparticles (but not right-handed neutrinos.)
- But these threshold corrections are all applied at M_{SUSY}

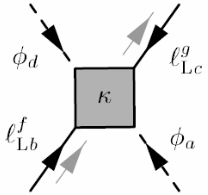
EFTs and the κ Matrix

- Seesaw formula not valid below seesaw scale

$$m_\nu = \frac{v^2}{2} Y_\nu^\dagger M_{N_R}^{-1} Y_\nu$$

- Right-handed neutrinos are decoupled and do not appear
- But evolution of ν_l is significant
- Use an EFT
- After integrating out each ν_r, replace it with an effective mass operator κ

EFTs: Diagram



$$: \quad i\kappa_{gf}\,\frac{1}{2}(\varepsilon^{cd}\varepsilon^{ba} + \varepsilon^{ca}\varepsilon^{bd})$$

Figure: The effective operator κ and its Feynman rule

Diagram from Antusch et al.

Decoupling Procedure

Neutrino Sector

1. Run down from M_{GUT} to largest M_{ν_R} eigenvalue.
2. (n.b. this eigenvalue runs.)
3. Rotate into mass eigenbasis (diagonalizing M_{ν_R}).
4. Zero this row/col of M_{ν_R} and this *row* of Y_ν.
5. Create the appropriate entries in κ.

Neutrino sector comparisons

- We compare Isajet-M to REAP.
- REAP is not a full-fledged code:
    - Doesn't search for unification
    - All parameters must be specified at the high scale
- So, run Isajet-M on a given input
- Extract relevant parameters at M_{GUT}
- Enter them into REAP, compare results

Neutrino comparisons

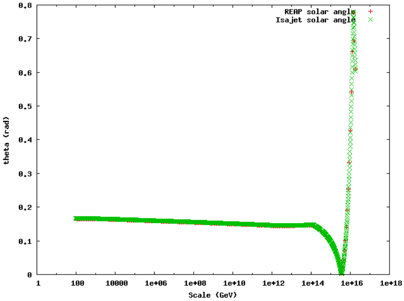


Figure: SPS 1b point, solar angle

$$Y_\nu(M_{\mathrm{GUT}}) = \mathrm{diag}(0.02, 0.1, 1.0)$$

$$M_N(M_{\mathrm{GUT}}) = \begin{pmatrix} -0.5554E+11 & -0.2205E+10 & -0.3448E+11 \\ -0.2205E+10 & -0.1433E+13 & -0.6204E+12 \\ -0.3448E+11 & -0.6204E+12 & -0.1461E+15 \end{pmatrix}$$

Neutrino comparisons

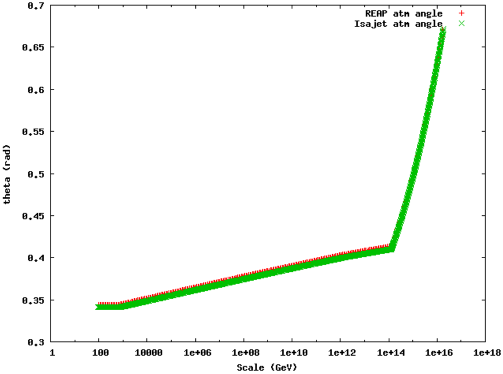


Figure: SPS 1b point, atmospheric angle

Neutrino comparisons

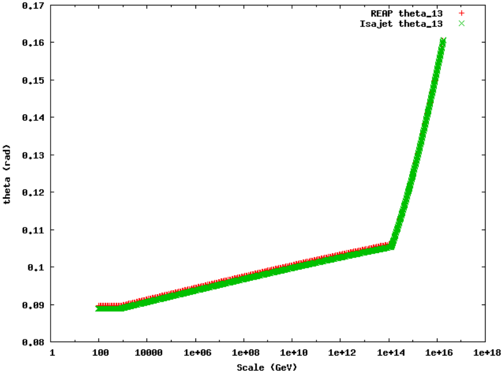


Figure: SPS 1b point, θ_{13}

Neutrino comparisons

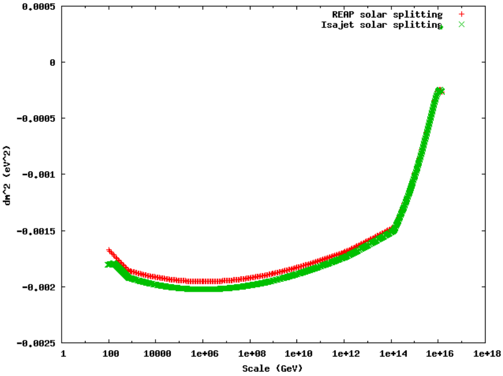


Figure: SPS 1b point, $\delta m^2_{\text{solar}}$

Neutrino comparisons

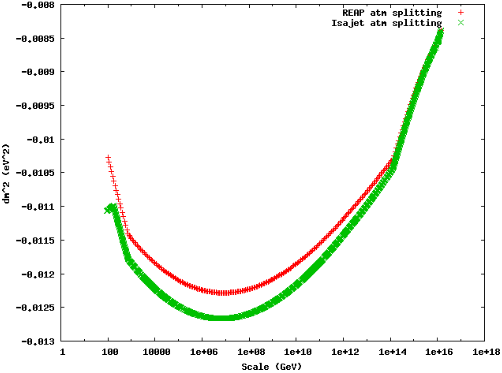


Figure: SPS 1b point, δm^2_{atm}

Neutrino comparisons

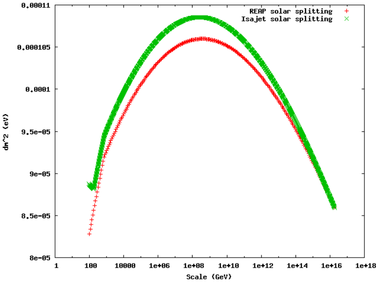


Figure: $\delta m^2_{\text{solar}}$

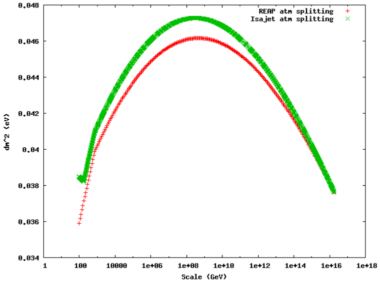


Figure: δm^2_{atm}

SPS1b point with $Y_\nu(M_{\mathrm{GUT}}) = \mathrm{diag}(0.001, 0.01, 0.1)$

$$M_N(M_{\mathrm{GUT}}) = \begin{pmatrix} 0.4070E+12 & 0.2320E+13 & -0.1990E+14 \\ 0.2320E+13 & -0.1350E+14 & 0.1160E+15 \\ -0.1990E+14 & 0.1160E+15 & -0.1000E+16 \end{pmatrix}$$

Neutrino comparisons

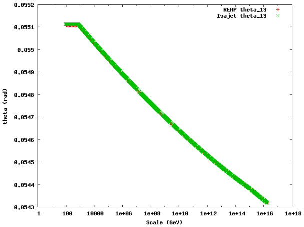


Figure: θ_{13}

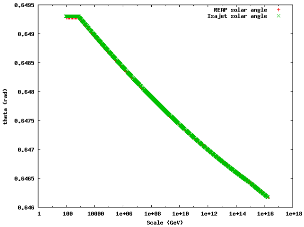


Figure: $\delta m^2_{\text{solar}}$

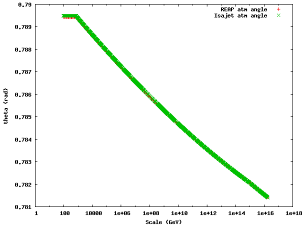


Figure: δm^2_{atm}

Neutrino mass/mixing comparison

SPS1a:

Code	θ_{12}	θ_{13}	θ_{23}	$m_{\nu 1}$ (eV)	$m_{\nu 2}$ (eV)	$m_{\nu 3}$ (eV)
SPS1a						
Isajet-M	0.142	0.105	0.41	0.219	0.216	0.192
REAP	0.1406	0.1058	0.4136	0.2114	0.2081	0.185
SPS1b						
Isajet-M	0.166	0.089	0.341	0.219	0.215	0.187
REAP	0.1654	0.08962	0.3444	0.2109	0.2069	0.1806
SPS2						
Isajet-M	0.142	0.105	0.411	0.213	0.21	0.186
REAP	0.1403	0.106	0.4144	0.2045	0.2014	0.179
SPS3						
Isajet-M	0.14	0.106	0.416	0.218	0.214	0.191
REAP	0.1382	0.1071	0.4193	0.2097	0.2066	0.1838
SPS4						
Isajet-M	0.195	0.056	0.209	0.214	0.209	0.169
REAP	0.1945	0.05647	0.2108	0.2062	0.2009	0.1629
SPS5						
Isajet-M	0.138	0.107	0.419	0.21	0.207	0.184
REAP	0.1366	0.1079	0.4227	0.2034	0.2004	0.1784

Comparison among codes

- At worst, we have a 5% difference from REAP.
- Similar differences exist for sparticle spectra among production codes:

Sparticle	Isajet	Isajet-M	Softsusy	Spheno	Suspect	(max-min)	diff[%]
χ_1^0	63.58	63.65	63.15	63.43	63.45	0.43	0.68
χ_2^0	116.63	116.4	116.92	117.00	117.09	0.46	0.39
χ_3^0	284.42	284.8	290.97	290.10	288.51	6.55	2.27
χ_4^0	305.78	306.5	307.15	306.56	307.57	1.79	0.58
$\chi_1^\pm$	116.08	115.9	116.61	116.16	116.11	0.53	0.45
$\chi_2^\pm$	304.50	305.3	305.92	308.77	308.55	4.27	1.39
$\tilde{u}_L$	397.88	397.94	394.65	398.50	397.19	3.84	0.97
$\tilde{u}_R$	388.28	388.38	387.80	388.56	387.43	1.13	0.29
$\tilde{\tau}_1$	95.57	94.14	95.35	95.30	94.55	1.02	1.07
$\tilde{\tau}_2$	154.54	151.31	155.18	154.98	154.59	3.23	2.1

$m_0 =$80 GeV, $m_{1/2} =$170 GeV, $A_0 =$-250 GeV, $\tan\beta = 10$, $\mu > 0$

Data from Kraml et al. automated comparison tool at `http://cern.ch/kraml/comparison/`.

Complex phases

- Complex phases in Y_ν can have enormous effect on the neutrino sector:

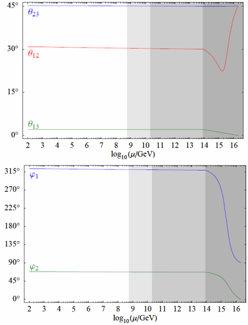


Figure 8: Highly non-linear running of θ_{12} and the Majorana phases in an example with large imaginary entries in the neutrino Yukawa matrix (see Eq. (29)). We used the MSSM with $\tan\beta = 10$, $M_{\mathrm{SUSY}} = 1\,\mathrm{TeV}$ and the following initial conditions at $M_{\mathrm{GUT}} = 2 \cdot 10^{16}\,\mathrm{GeV}$: $\theta_{12} = \theta_{23} = \pi/4$, $\theta_{13} = 0$, $\varphi_1 = \pi/2$, $\varphi_2 = 0$, normal hierarchy, $m_1 = 0.08\,\mathrm{eV}$, $\Delta m^2_{\mathrm{sol}} = 1.1 \cdot 10^{-4}\,\mathrm{eV}^2$, $\Delta m^2_{\mathrm{atm}} = 4 \cdot 10^{-3}\,\mathrm{eV}^2$.

- They are unconstrained
- We are now complexifying Isajet-M

Searches in parameter space

- Goal: find models that match all observable data
- Simple ansatz at M_{GUT} for Y's, M_N
- Choose grid on parameter space, test points

Summary

- To analyze GUTs, we have to solve the β-functions with mixed boundary conditions.
- To understand flavor physics and neutrino mixings, we need to evolve the couplings in full matrix form.

Future outlook

- Analyze parameter space for various SUSY GUTs, looking for viable points.
- Complexify and add analysis of baryogenesis/leptogenesis constraints.

- R Dermisek and S Raby.
  Bi-large neutrino mixing and CP violation in an SO(10) SUSY GUT for fermion masses.
  Phys Lett B, 2005.
  `hep-ph/0507045`
- S Antusch, J Kerseten, M Lindner, M Ratz, MA Schmidt.
  Running neutrino mass parameters in see-saw scenarios.
  JHEP, 2005.
  `arXiv:hep-ph/0501272`
- S Antusch, M Drees, J Kersten, M Lindner, M Ratz.
  Neutrino Mass Operator Renormalization Revisited
  Phys Lett B519:238, 2001.
  `arXiv:hep-ph/0108005`
- D Pierce, J Bagger, K Matchev, R Zhang.
  Precision corrections in the minimal supersymmetric standard model.
  Nucl Phys B, 1997.
  `arXiv:hep-ph/9606211`