A Pseudo-Goldstone Boson Approach for Dealing with Little Hierarchy Problems

Benjamin N. Grossmann

Oklahoma State University

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(in collaboration with Dr. Z. Tavartkiladze and Dr. S. Nandi.)

Outline

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- Hierarchy Problems
  - Little Hierarchy Problems in SUSY
- Global Symmetry and Pseudo-Goldstone Bosons
- The Model
  - Symmetries
  - States
  - Parameters
- Effect on the Higgs Mass
- Conclusion

Standard Model: Higgs Sector

- The potential: $V = -m^2|h|^2 + \lambda|h|^4$
- Symmetry breaking and the vacuum expectation value:

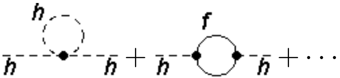
$$\langle h \rangle = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix} \longrightarrow v^2 = \frac{m^2}{\lambda} \sim (174 \text{ GeV})^2$$

- This is fine at tree level.
- However, when loop diagrams are included, things change...

Hierarchy Problems

Standard Model: Loops

Loop Corrections to the Higgs Mass

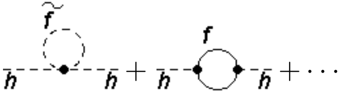


- One-loop Higgs mass corrections:

$$\Delta m^2 \sim \frac{\lambda^2}{(4\pi)^2}\Lambda^2$$

  - There are also corrections due to all the other particles.
- This is satisfactory up to a cutoff of $\Lambda \sim 1$ TeV
- Without new physics, the cutoff scale Λ should be as high as the Plank mass, $\Lambda \sim 10^{16}$ TeV.
  - This requires fine-tuning on the order of 10^{-32}.
- Low scale Supersymmetry provides a solution.

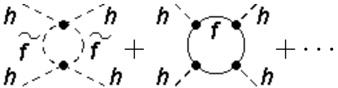
Super particles lessen the divergences.



$$\frac{\lambda^2}{(4\pi)^2}\Lambda^2$$
$$\downarrow$$
$$\frac{3\lambda_t^2}{4\pi^2}\, m_{\tilde{t}}^2 \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right)$$

- Naturalness suggests $m_{\tilde{t}}$ should be light. ($\sim$ 1 TeV)

However, there are finite corrections due to the quartic coupling.



$$m_h^2 = M_Z^2 \cos^2(2\beta) + \frac{6g^2 m_t^4}{(4\pi)^2 M_W^2} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

- To exceed LEP bound ($m_{h^0} > 114$ GeV), $m_{\tilde{t}}$ should be large.

$$M_z^2 = -2(m_{h_u}^2 + |\mu|^2 + \frac{2}{\tan^2\beta}(m_{h_d}^2 - m_{h_u}^2))$$

- The values of m_{h_u} and $|\mu|$ are $\sim$1 TeV.
- But for $M_Z = 91 GeV$ they must be fine-tuned for the difference of their squares to be $\sim M_Z^2$.

Fine-tuning is required on the order of $\sim 10^{-2}$.
This is much better than the SM, but not entirely satisfactory.

Propose to protect the Higgs mass by treating the MSSM Higgs doublets $(\hat{h}_u, \hat{h}_d)$ as pseudo Goldstone bosons.

Consider the scalar sector of MSSM + new $SU(2)_L$ Higgs singlets:

$$(\hat{h}_u, \hat{h}_d) \text{ and } (\hat{S}_u, \hat{S}_d, \hat{X})$$

The superpotential can be written as

$$W_{\text{Higgs}} = \kappa \hat{X} \hat{h}_u \hat{h}_d + \kappa' \hat{X} \hat{S}_u \hat{S}_d - \kappa' \hat{X} M^2$$

Suppose $\kappa = \kappa'$ at some high scale Λ', then

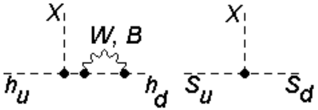
$$W_{\text{Higgs}(\Lambda')} = \kappa \hat{X} (\hat{h}_u \hat{h}_d + \hat{S}_u \hat{S}_d) - \kappa' \hat{X} M^2$$

$$W_{\text{Higgs}(\Lambda')} = \kappa \hat{X}(\underbrace{\hat{h}_u \hat{h}_d + \hat{S}_u \hat{S}_d}_{SU(3)\text{ symmetry}}) - \kappa' \hat{X} M^2$$

- Symmetry of the superpotential has increased.
- Global symmetry $SU(3)_G$ has triplets $(\hat{h}_u, \hat{S}_u)$ and $(\hat{h}_d, \hat{S}_d)$.
- Pseudo-Goldstone bosons are generated by symmetry breaking

$$SU(3)_G \xrightarrow{\langle S_u \rangle, \langle S_d \rangle, \langle X \rangle} SU(2)_G.$$
$$8 - 3 = 5 = \underset{h_u}{2} + \underset{h_d}{2} + 1 \text{ (similar to NMSSM)}$$

$SU(2)_L \times U(1)_Y$ violate the approximate global symmetry $SU(3)_G$.



Extending The Global Symmetry
to the Yukawa Sector

- $SU(3)_{\text{color}}$ and Yukawa interactions (especially λ_t) can be made to respect $SU(3)_G$.
- Extension: an $SU(2)_L$ singlet vector-like $\hat{U}_i$ (and $\hat{U}_i^c$).

Then in the Yukawa sector, the top Yukawa interactions of the superpotential look like:

$$W_{\text{Yukawa}} = \lambda_t \hat{u}_3^c \hat{h}_u \hat{q}_3 + \lambda_t' \hat{u}_3^c \hat{S}_u \hat{U}_3$$

Again, choose $\lambda = \lambda'$ at the scale Λ'.
And the triplet $(\hat{q}_3, \hat{U}_3)$ can be formed, such that:

$$W_{\text{Yukawa}(\Lambda')} = \lambda_t \hat{u}_3^c (\underbrace{\hat{h}_u \hat{q}_3 + \hat{S}_u \hat{U}_3}_{SU(3)_G \text{ symmetry}})$$

- g_3 and λ_t respect $SU(3)_G$

The relationships between the couplings in the superpotential can be characterize in terms of their relative differences.

$$\frac{\kappa' - \kappa}{\kappa'} = \delta_\kappa \leftrightarrow \frac{\kappa}{\kappa'} = 1 + \delta_\kappa \qquad \frac{\lambda_t}{\lambda_t'} = 1 + \delta_\lambda$$

- At the scale Λ', $\delta_i = 0$. Below the scale Λ', $|\delta_i| \lesssim \frac{1}{4}$.
- Use the superpotential to find the appropriate terms in the potential. $V = V_F + V_D + V_{\text{soft}}$
- From V_{soft}, more parameter relationships can be constructed:

$$\frac{\kappa}{\kappa'}, \frac{\lambda_t}{\lambda_t'}, \frac{A_\kappa}{A_\kappa'}, \frac{A_{\lambda_t}}{A_{\lambda_t}'}, \frac{m_{h_u}^2}{m_{S_u}^2}, \frac{m_{h_d}^2}{m_{S_d}^2}, \frac{m_{\widetilde{q}}^2}{m_{\widetilde{U}}^2} \sim (1 + \delta_i)$$

22 Parameters—22 Equations

$$\begin{pmatrix} \kappa & \lambda_t \\ \kappa' & \lambda'_t \end{pmatrix} \begin{pmatrix} A_\kappa & A_{\lambda_t} \\ A'_\kappa & A'_{\lambda_t} \end{pmatrix} \begin{pmatrix} m^2_{h_u} & m^2_{h_d} & m^2_{\tilde{q}} \\ m^2_{S_u} & m^2_{S_d} & m^2_{\tilde{U}} \end{pmatrix} \begin{pmatrix} m^2_{\tilde{u}^c} & m^2_X \end{pmatrix}$$

$$\begin{pmatrix} \alpha_1 & \alpha_2 & \alpha_3 \end{pmatrix} \quad \text{gauge couplings}$$

$$\begin{pmatrix} M_1 & M_2 & M_3 \end{pmatrix} \quad \text{gaugino masses}$$

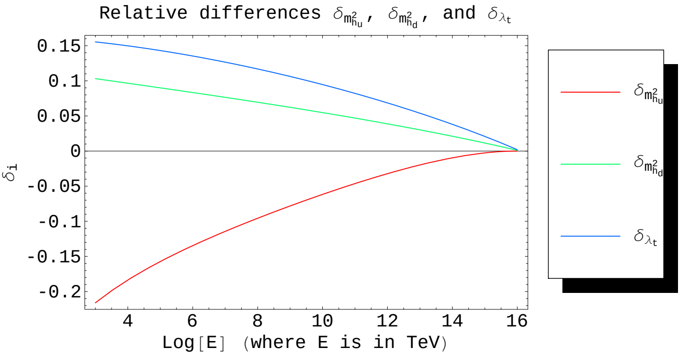
One Loop RGE of the λ_t and λ'_t Couplings

$$\frac{\partial}{\partial x}\lambda_t = \frac{\lambda_t}{(4\pi)^2}\left(6\lambda_t^2 + \lambda_t'^2 + \kappa^2\right) - \frac{\lambda_t}{4\pi}\left(\frac{16}{3}\alpha_3 + 3\alpha_2 + \frac{13}{15}\alpha_1\right)$$

$$\frac{\partial}{\partial x}\lambda'_t = \frac{\lambda'_t}{(4\pi)^2}\left(5\lambda_t'^2 + 2\lambda_t^2 + \kappa'^2\right) - \frac{\lambda'_t}{4\pi}\left(\frac{16}{3}\alpha_3 + \frac{16}{15}\alpha_1\right)$$

Model Parameters

- Boundary conditions: All $\delta_i = 0$ at $\Lambda' \to M_{GUT}$.
- The $|\delta_i| \lesssim \frac{1}{5}$.



Mass of the Higgs

The Higgs mass matrix (h_d^0 and h_u^0) resembles that of the MSSM.

$$M^2_{h(doublets)} = \begin{pmatrix} m_1^2 & m_3^2 \\ m_3^2 & m_2^2 \end{pmatrix} \qquad\qquad m^2_{1,2,3} \sim m_0^2 \sim (\text{few TeV})^2$$

- We have $\text{Tr}(M^2_{h(doublets)}) = m_0^2$.
  and
- Due to $SU(3)_G$ breaking: $\det(M^2_{h(doublets)}) \simeq \delta_0 m_0^4$.

$\Rightarrow m^2_{h\text{-lighter}} \approx \delta_0 m_0^2$ with $\delta_0 \sim \frac{1}{10}$ (natural value)

The loop corrections to the Higgs mass become modified.

$$\frac{3\lambda_t^2}{4\pi^2} m_{\tilde{t}}^2 \delta \log\left(\frac{\Lambda}{m_{\tilde{t}}}\right) \qquad \frac{6g^2 m_t^4}{(4\pi)^2 M_W^2} \log\left(\frac{m_{\tilde{t}}^2}{m_t^2}\right)$$

- For a $\delta \sim \frac{1}{10}$, then the stop mass can be increased accordingly, without additional fine-tuning.

Conclusion

- SUSY remedies the Standard Model Λ^2 divergences
  - MSSM has a "Little" Hierarchy Problem
- Possible solution based on Pseudo-Goldstone Bosons
  - Global symmetry $SU(3)_G$ breaks, creates PGBs
  - MSSM Higgs doublets act as the PBGs
- Constraints are relaxed on the physical Higgs mass