Neutrino Mass Models

- Neutrino mass and mixing
- Neutrino mass models roadmap (survey)
- Family symmetry
- GUT relations and predictions
- Implications for the LHC

Steve King, Pheno'08, University of Wisconsin, Madison, 28th April, 2008

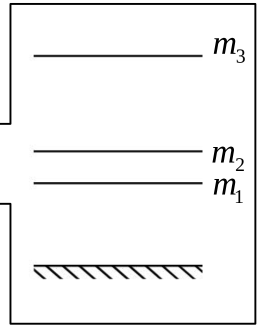
UNIVERSITY OF Southampton
School of Physics and Astronomy

Neutrino Mass and Mixing

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$$

Standard Model states

Neutrino mass states



$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{MNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23}e^{-i\delta_{23}} \\ 0 & -s_{23}e^{i\delta_{23}} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12}e^{-i\delta_{12}} & 0 \\ -s_{12}e^{i\delta_{12}} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Atmospheric — Reactor — Solar

Oscillation phase $\delta = \delta_{13} - \delta_{23} - \delta_{12}$

3 masses + 3 angles + 1 phase
= 7 new parameters for SM

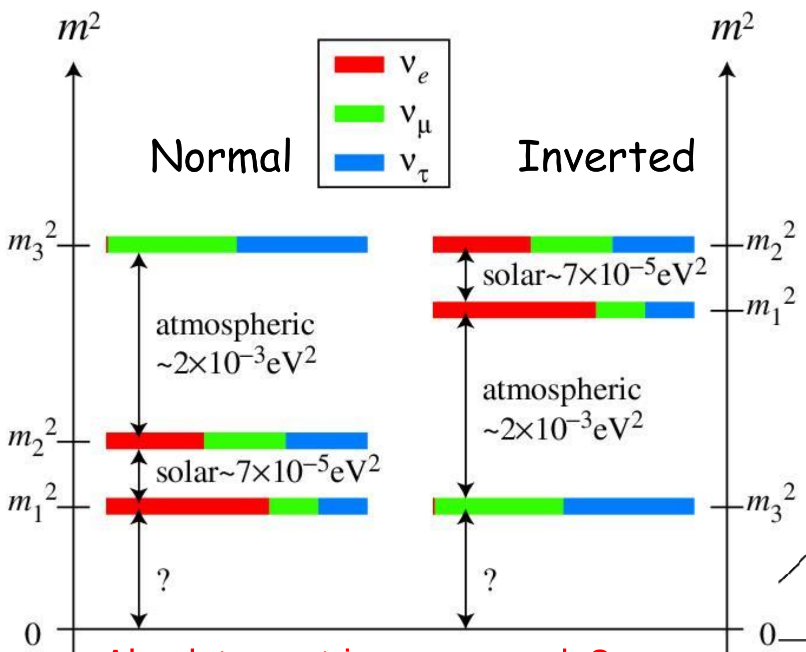
Neutrino mass squared splittings and angles

parameter	best fit	3σ range
$\Delta m^2_{21}\ [10^{-5}\ \mathrm{eV}^2]$	7.9	7.1–8.9
$\Delta m^2_{31}\ [10^{-3}\ \mathrm{eV}^2]$	2.6	2.0–3.2

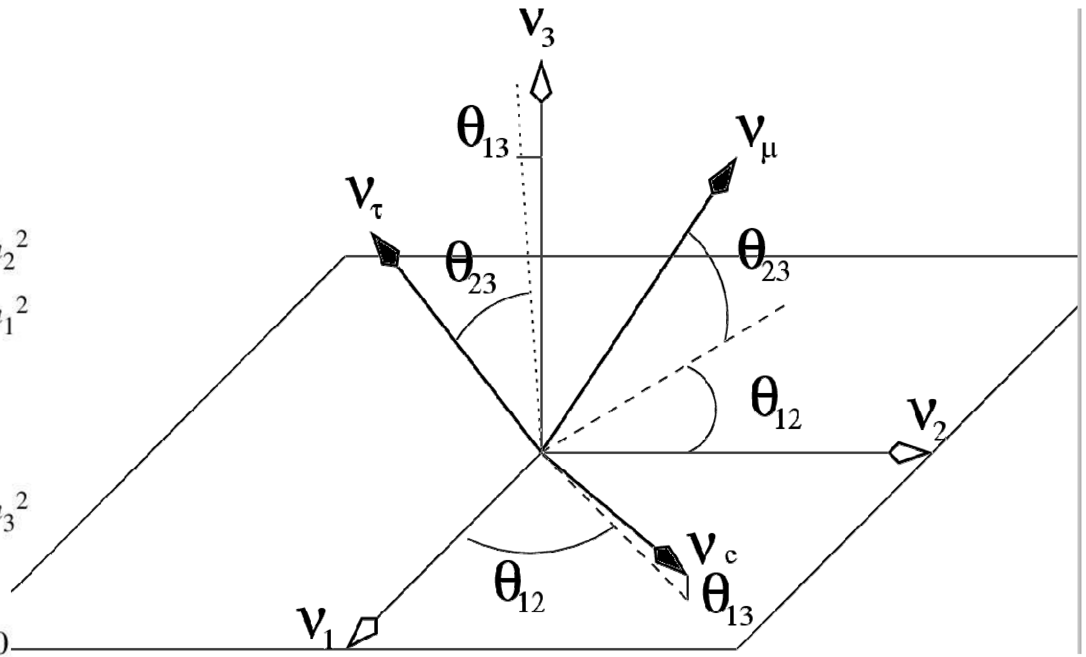
Valle et al

$$\theta_{12} = 33^\circ \pm 5^\circ$$
$$\theta_{23} = 45^\circ \pm 10^\circ$$
$$\theta_{13} < 13^\circ$$

3 σ errors



Absolute neutrino mass scale?



Tri-bimaximal mixing (TBM)

Harrison, Perkins, Scott

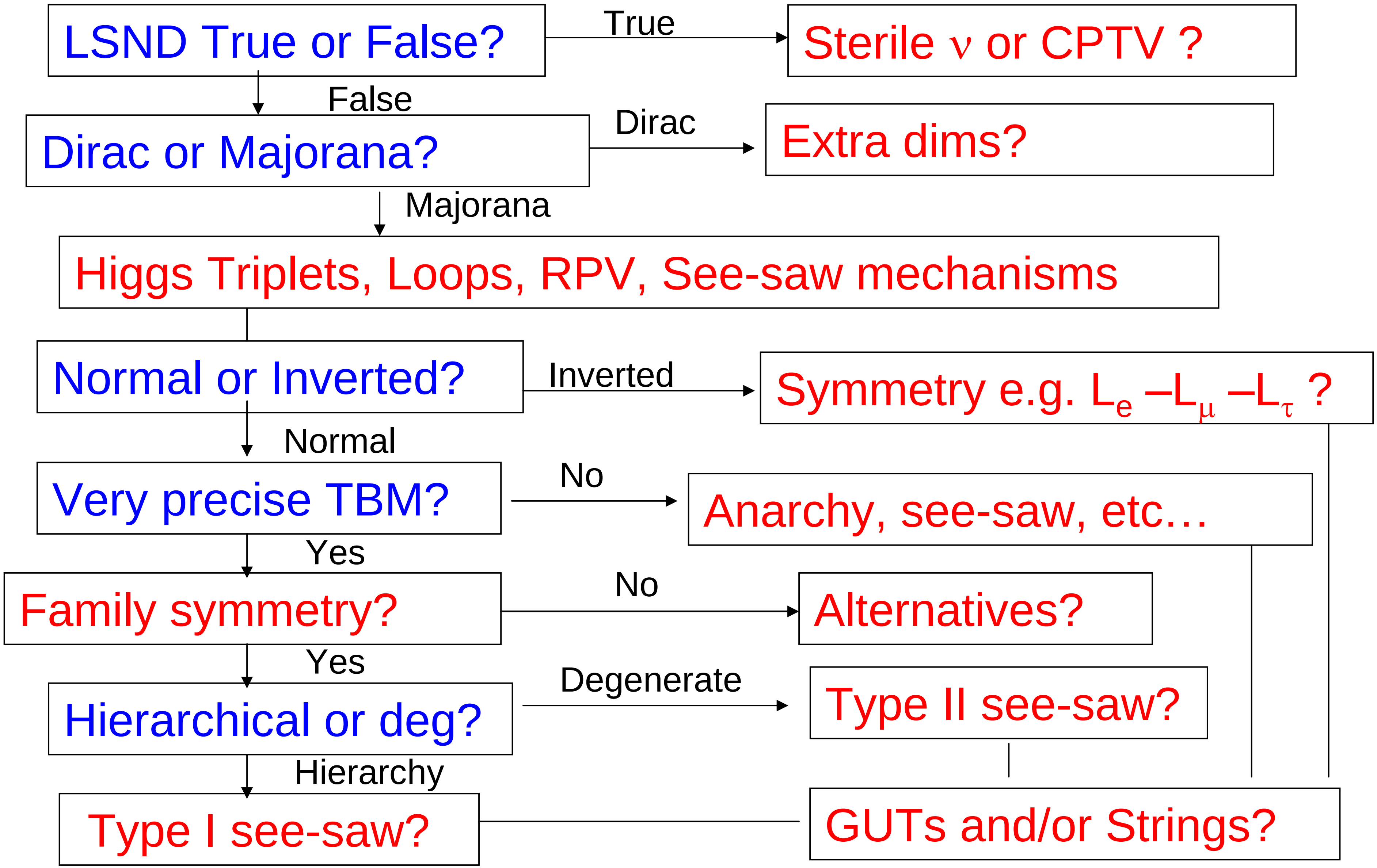
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\theta_{12} = 35^\circ, \qquad \theta_{23} = 45^\circ, \qquad \theta_{13} = 0^\circ.$$

c.f. data $\theta_{12} = 33^\circ \pm 5^\circ, \theta_{23} = 45^\circ \pm 10^\circ, \; \theta_{13} < 13^\circ$

- Current data is consistent with TBM
- But no convincing reason for exact TBM – expect deviations

Neutrino mass models roadmap



Why not Standard model?

1. There are no right-handed neutrinos ν_R
2. There are only Higgs doublets of $SU(2)_L$
3. There are only renormalizable terms

In the Standard Model these conditions all apply so neutrinos are massless, with ν_e , ν_μ , ν_τ distinguished by separate lepton numbers L_e, L_μ, L_τ

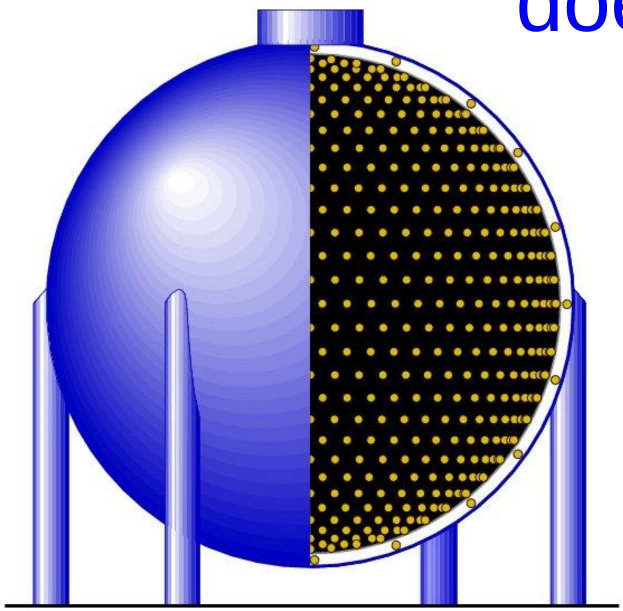
Neutrinos and anti-neutrinos are distinguished by the total conserved lepton number $L=L_e+L_\mu+L_\tau$

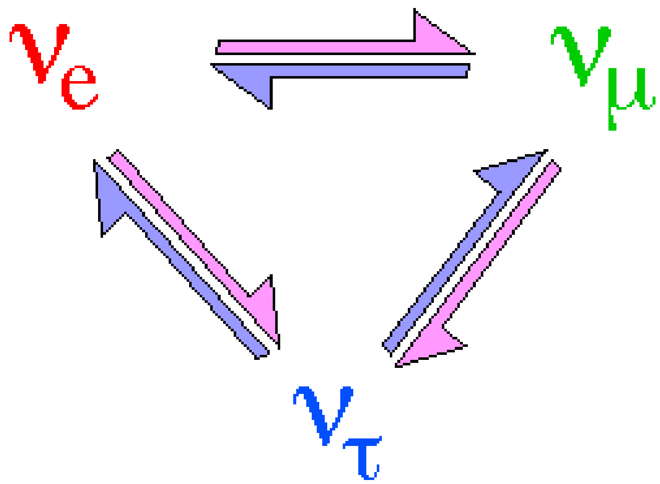
To generate neutrino mass we must relax 1 and/or 2 and/or 3

LSND True or False?

MiniBoone does not support LSND result

does support three neutrinos





In this talk we assume that LSND is false

Dirac or Majorana?

CP conjugate

Majorana masses

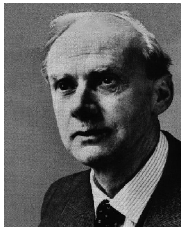
$$m_{LL}\bar{\nu}_L \nu_L^c$$

$$M_{RR}\bar{\nu}_R \nu_R^c$$

Violates L

Violates L_e, L_μ, L_τ

Neutrino=antineutrino



$$m_{LR}\bar{\nu}_L \nu_R$$

Conserves L

Violates L_e, L_μ, L_τ

Neutrino $\neq$ antineutrino

Dirac mass

Dirac

Recall origin of electron mass in SM with $L = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L , \quad e_R^-, \quad H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$

$$\lambda_e \overline{L} H e_R^- = \lambda_e \langle H^0 \rangle \overline{e_L^-} e_R^-$$

Yukawa coupling λ_e must be small since $\langle H^0 \rangle = 175$ GeV

$$m_e = \lambda_e \langle H^0 \rangle \approx 0.5\, MeV \Leftrightarrow \lambda_e \approx 3.10^{-6}$$

Introduce right-handed neutrino ν_{eR} with zero Majorana mass

$$\lambda_\nu \overline{L} H^c \nu_{eR} = \lambda_\nu \langle H^0 \rangle \overline{\nu}_{eL} \nu_{eR}$$

then Yukawa coupling generates a Dirac neutrino mass

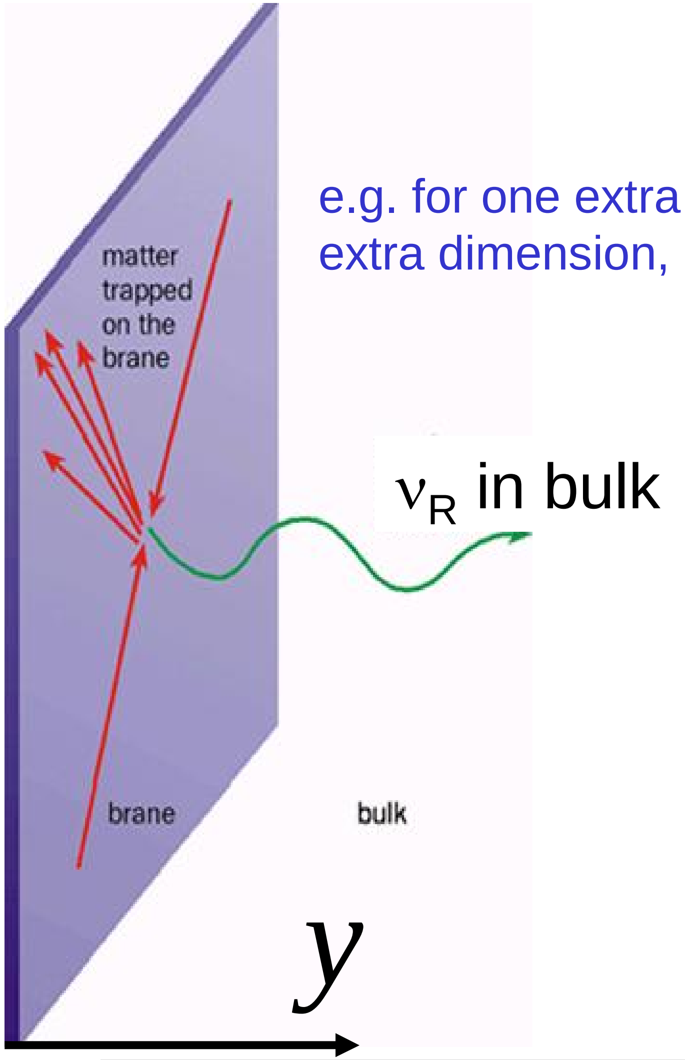
$$m_{LR}^\nu = \lambda_\nu \langle H^0 \rangle \approx 0.2\, eV \Leftrightarrow \lambda_\nu \approx 10^{-12}$$

Why so small?
– extra dimensions

Flat extra dimensions with RH neutrinos in the bulk

$$M_{Planck}^{2} = M_{string}^{2+\delta} R^{\delta}$$

Number of extra dimensions



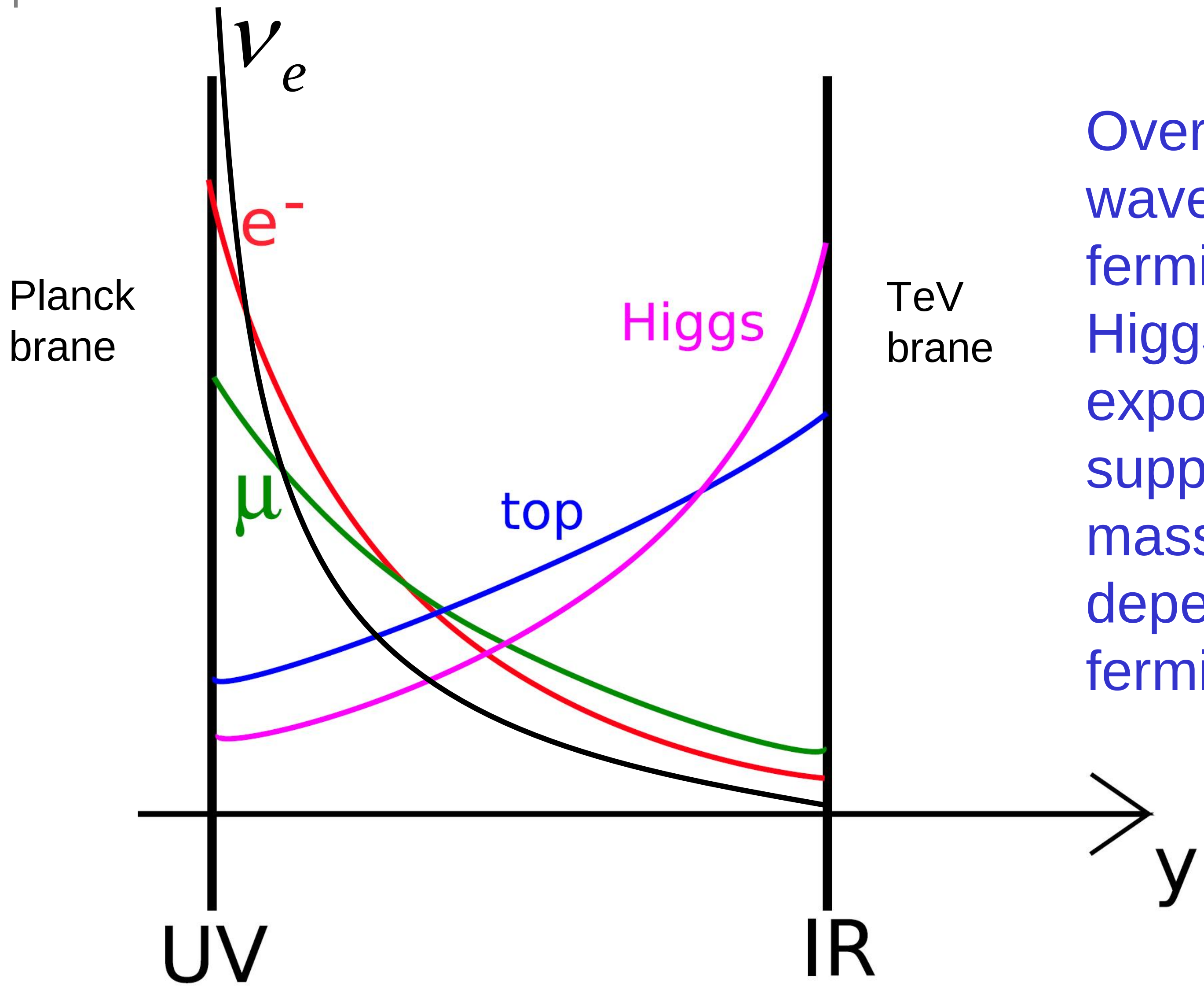
e.g. for one extra dimension y the ν_R wavefunction spreads out over the extra dimension, leading to a volume suppressed Yukawa coupling at y=0

$$\lambda \int d^4x H(x) L(x) \nu_R(x, y=0)$$

$$\rightarrow m_{LR}^{\nu} = \frac{\lambda \langle H^0 \rangle}{\sqrt{V}} = \lambda \langle H^0 \rangle \frac{M_{string}}{M_{Planck}}$$

$$e.g. \quad \frac{M_{string}}{M_{Planck}} = \frac{10^7}{10^{19}} = 10^{-12}$$

Warped extra dimensions with SM in the bulk



Overlap wavefunction of fermions with Higgs gives exponentially suppressed Dirac masses, depending on the fermion profiles

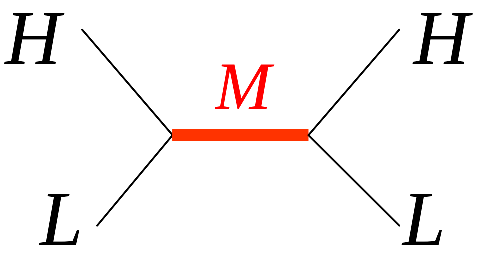
Majorana

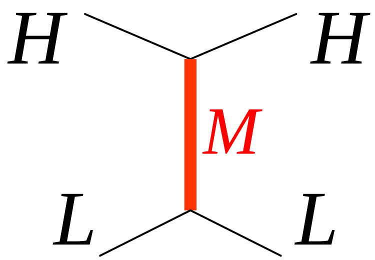
Renormalisable
$\Delta L=2$ operator $\lambda_\nu LL\Delta$ where Δ is light Higgs triplet with VEV < 8GeV from ρ parameter

Non-renormalisable
$\Delta L=2$ operator $\frac{\lambda_\nu}{M}LLHH=\frac{\lambda_\nu}{M}\langle H^0\rangle^2\,\overline{\nu}_{eL}\nu_{eL}^c$ Weinberg

This is nice because it gives naturally small Majorana neutrino masses $m_{LL}\sim \langle H^0\rangle^2/M$ where M is some high energy scale

The high mass scale can be associated with some heavy particle of mass M being exchanged (can be singlet or triplet)



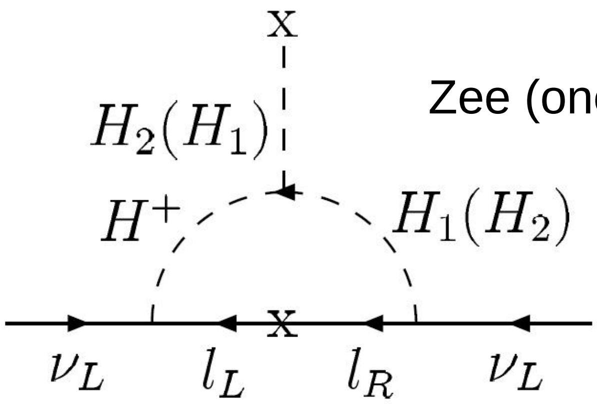


- Loop models
- RPV SUSY
- See-saw mechanisms

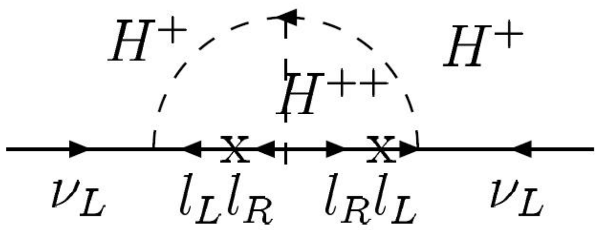
- Loop models

Introduce Higgs singlets and triplets with couplings to leptons

$$-\mathcal{L}^{yuk} = f_{ij} H^{++} l_i l_j + g_{ij} H^{+} l_i \nu_j + h_{ij} H^0 \nu_i \nu_j$$



Zee (one loop)



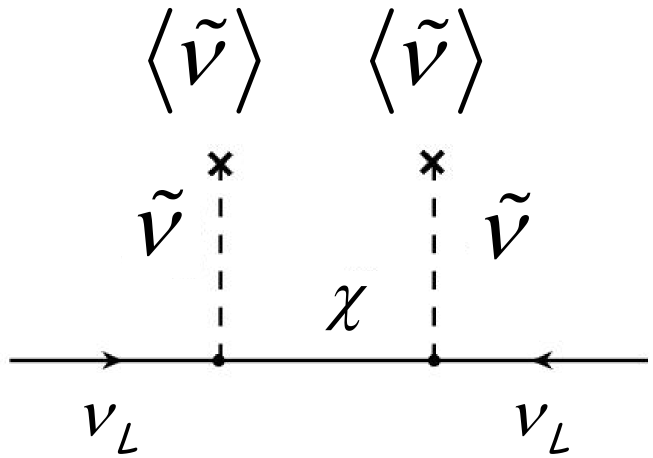
Babu (two loop)

- RPV SUSY

Another way to generate Majorana masses is via SUSY

Scalar partners of lepton doublets (slepton doublets) have same quantum numbers as Higgs doublets

If R-parity is violated then sneutrinos may get (small) VEVs inducing a mixing between neutrinos and neutralinos χ

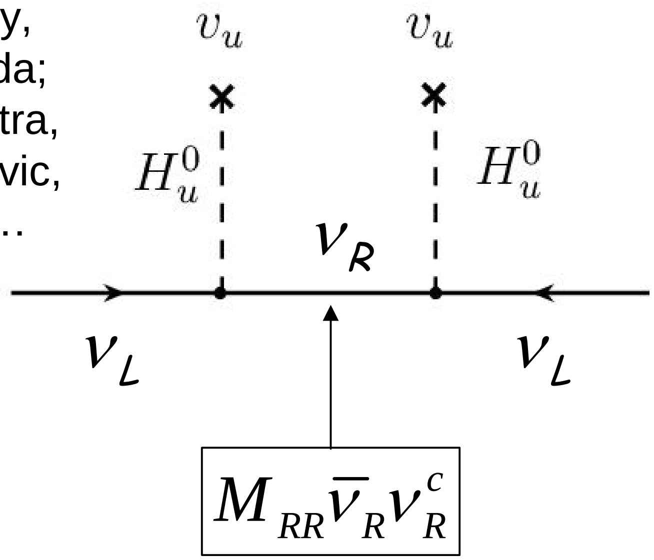


$$m_{LL}^{\nu} \approx \frac{\langle \tilde{\nu} \rangle^2}{M_{\chi}} \approx \frac{MeV^2}{TeV} \approx eV$$

•Types of see-saw mechanism

P.Minkowski, Gell-Mann, Ramond, Slansky, Yanagida; Mohapatra, Senjanovic, Valle,…

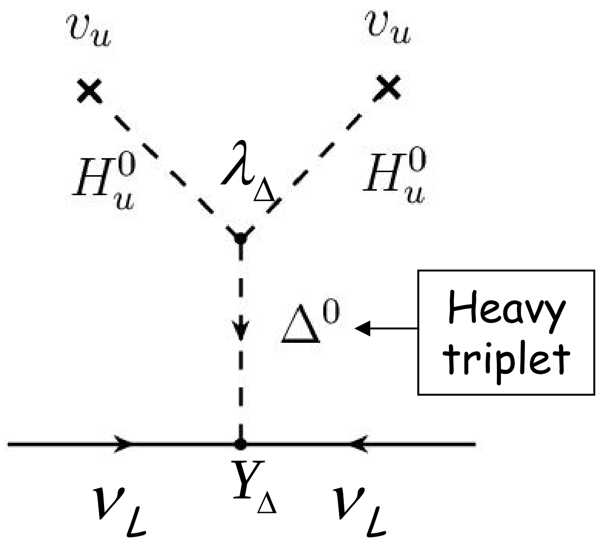
Type I see-saw mechanism



$$m_{LL}^{I} \approx -m_{LR} M_{RR}^{-1} m_{LR}^{T}$$

Type II see-saw mechanism

Lazarides, Magg, Mohapatra, Senjanovic, Shafi, Wetterich (1981)



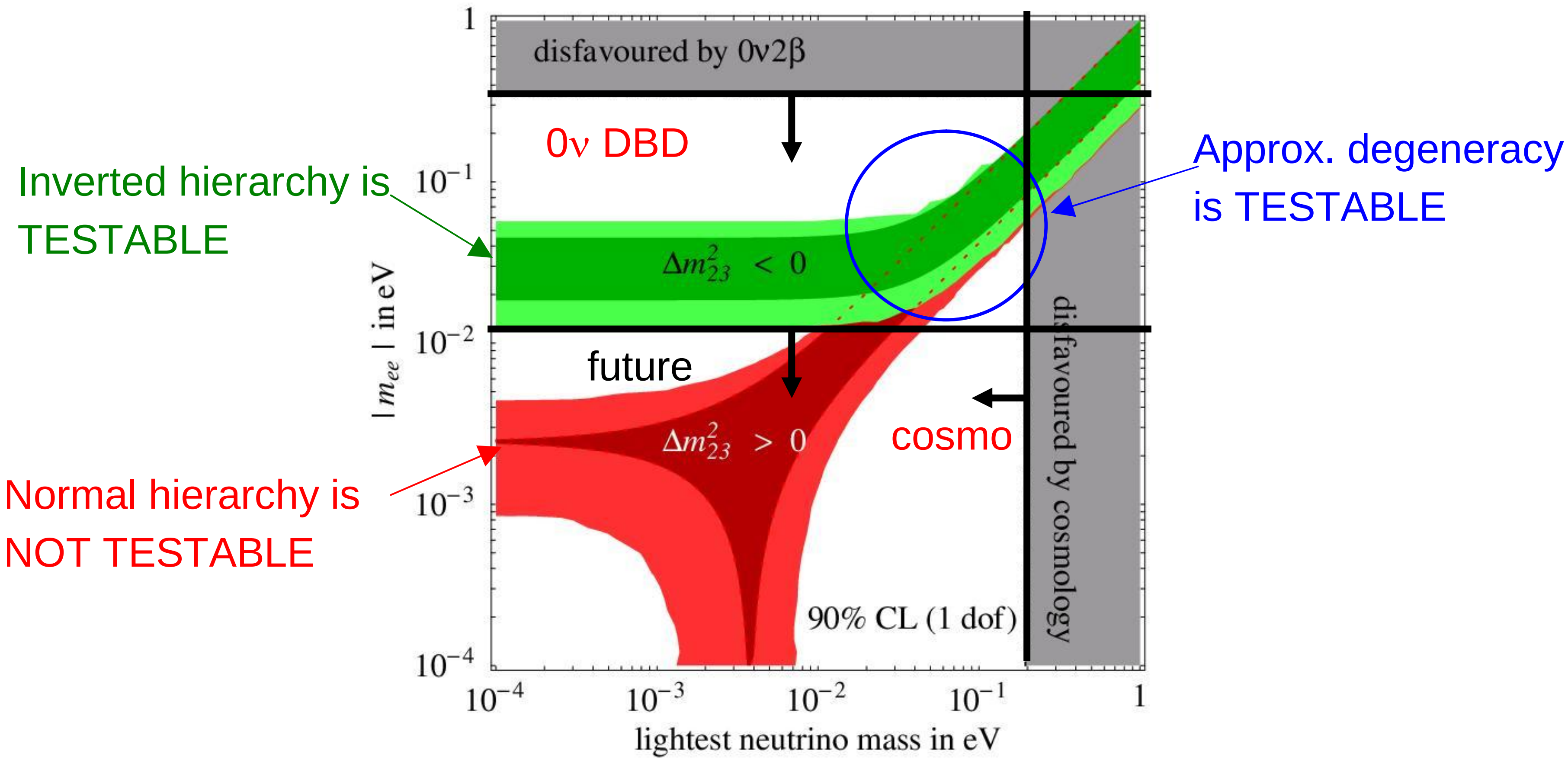
$$m_{LL}^{II} \approx \lambda_\Delta Y_\Delta \frac{v_u^2}{M_\Delta}$$

Normal or Inverted or Degenerate?

	Type A (zero in 11)	Type B (non-zero 11)
Normal Hierarchy	$m_{LL}^{HI} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \frac{m}{2}$	
Inverted hierarchy	$m_{LL}^{IH(A)} \approx \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \frac{m}{\sqrt{2}}$	$m_{LL}^{IH(B)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/2 & 1/2 \\ 0 & 1/2 & 1/2 \end{pmatrix} m$
Degenerate	$m_{LL}^{DEG(A)} \approx \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & \frac{1}{2} & \frac{1}{2} \end{pmatrix} m$	$m_{LL}^{DEG(B1)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} m$ $m_{LL}^{DEG(B2)} \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} m$

Pseudo-Dirac

Inverted or degenerate can be tested by neutrinoless double beta decay



from: F. Feruglio, A. Strumia, F. Vissani ('02)

How precise is Tri-bimaximal mixing?

To answer this it is useful to parametrize the PMNS mixing matrix in terms of deviations from TBM

SFK

$$s_{13} = \frac{r}{\sqrt{2}}, \quad s_{12} = \frac{1}{\sqrt{3}}(1+s), \quad s_{23} = \frac{1}{\sqrt{2}}(1+a)$$

$$0 < r < 0.22, \; -0.11 < s < 0.04, \; -0.12 < a < 0.13.$$

r = reactor s = solar a = atmospheric

$$U \approx \begin{pmatrix} \sqrt{\frac{2}{3}}(1-\frac{1}{2}s) & \frac{1}{\sqrt{3}}(1+s) & \frac{1}{\sqrt{2}}re^{-i\delta} \\ -\frac{1}{\sqrt{6}}(1+s-a+re^{i\delta}) & \frac{1}{\sqrt{3}}(1-\frac{1}{2}s-a-\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1+a) \\ \frac{1}{\sqrt{6}}(1+s+a-re^{i\delta}) & -\frac{1}{\sqrt{3}}(1-\frac{1}{2}s+a+\frac{1}{2}re^{i\delta}) & \frac{1}{\sqrt{2}}(1-a) \end{pmatrix}$$

If r,s,a are very small this probably requires family symmetry

Sequential Dominance

SFK 98- (Basis Invariant '06)

columns

Diagonal RH nu basis

$$M_{\rm RR} = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & Z \end{pmatrix}$$

$$Y^{\nu}_{\rm LR} = \begin{pmatrix} A & B & C \end{pmatrix}$$

See-saw ⇨

$$m^{\nu}_{LL} = \frac{AA^T}{X} + \frac{BB^T}{Y} + \frac{CC^T}{Z}$$

Sequential dominance ⇨ Dominant m_3 Subdominant m_2 Decoupled m_1

$$\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^{\dagger}B &= 0 \end{aligned}$$

Constrained SD

⇨

$$V^{\nu_L \dagger} \approx \begin{pmatrix} -\sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Tribimaximal HPS

This implies a non-Abelian family symmetry

Need

$$Y_{LR}^{\nu} = \begin{pmatrix} 0 & B_1 & - \\ A_2 & B_2 & - \\ A_3 & B_3 & - \end{pmatrix}$$

with

$$\begin{aligned} |A_1| &= 0, \\ |A_2| &= |A_3|, \\ |B_1| &= |B_2| = |B_3|, \\ A^\dagger B &= 0 \end{aligned}$$

$2 \leftrightarrow 3$ symmetry (from maximal atmospheric mixing)

$1 \leftrightarrow 2 \leftrightarrow 3$ symmetry (from tri-maximal solar mixing)

Examples of suitable non-Abelian Family Symmetries:

SFK, Ross; Velasco-Sevilla; Varzelias $SU(3)$ Δ_{27}

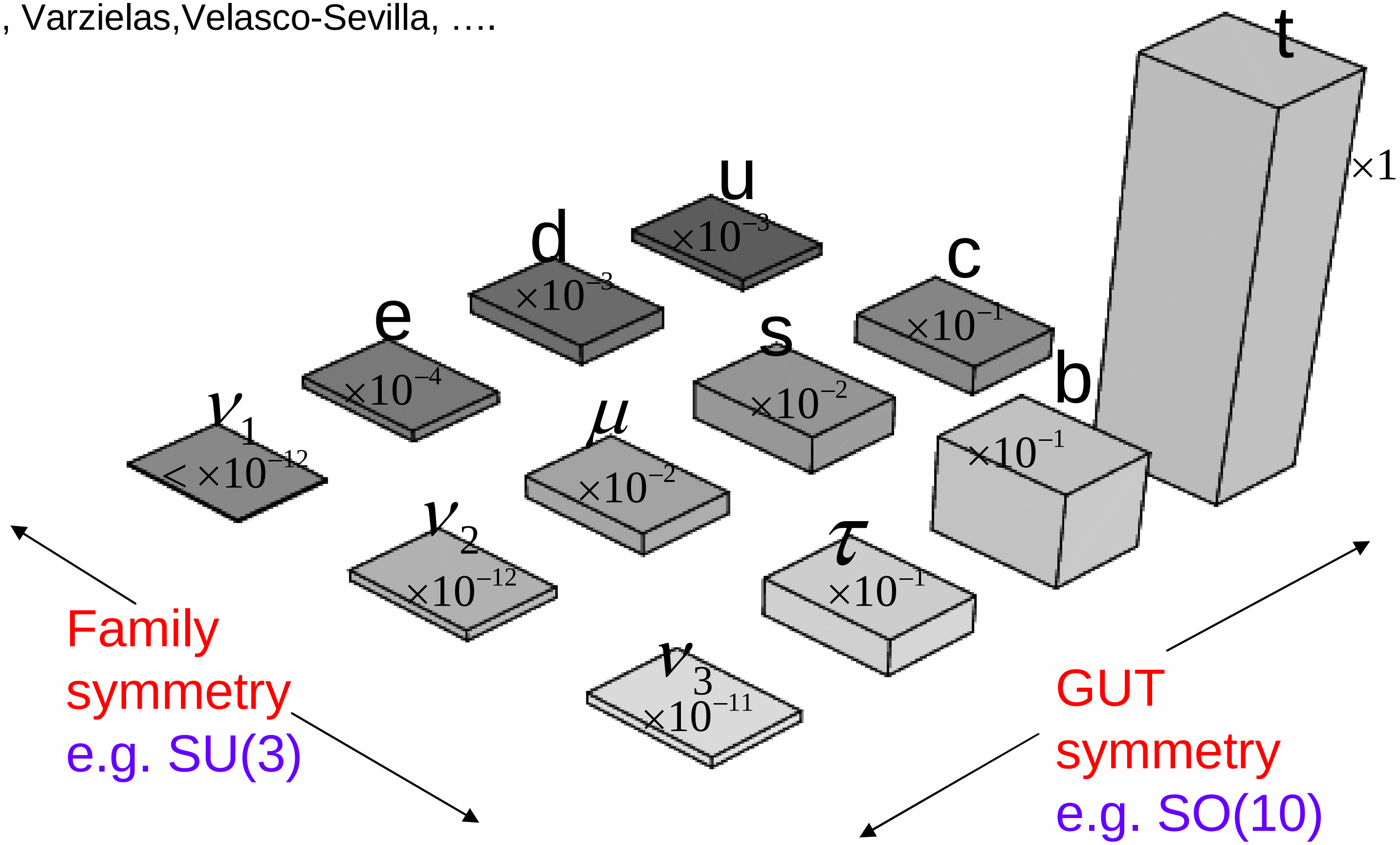
SFK, Malinsky $SO(3)$ A_4

} Discrete subgroups preferred by vacuum alignment

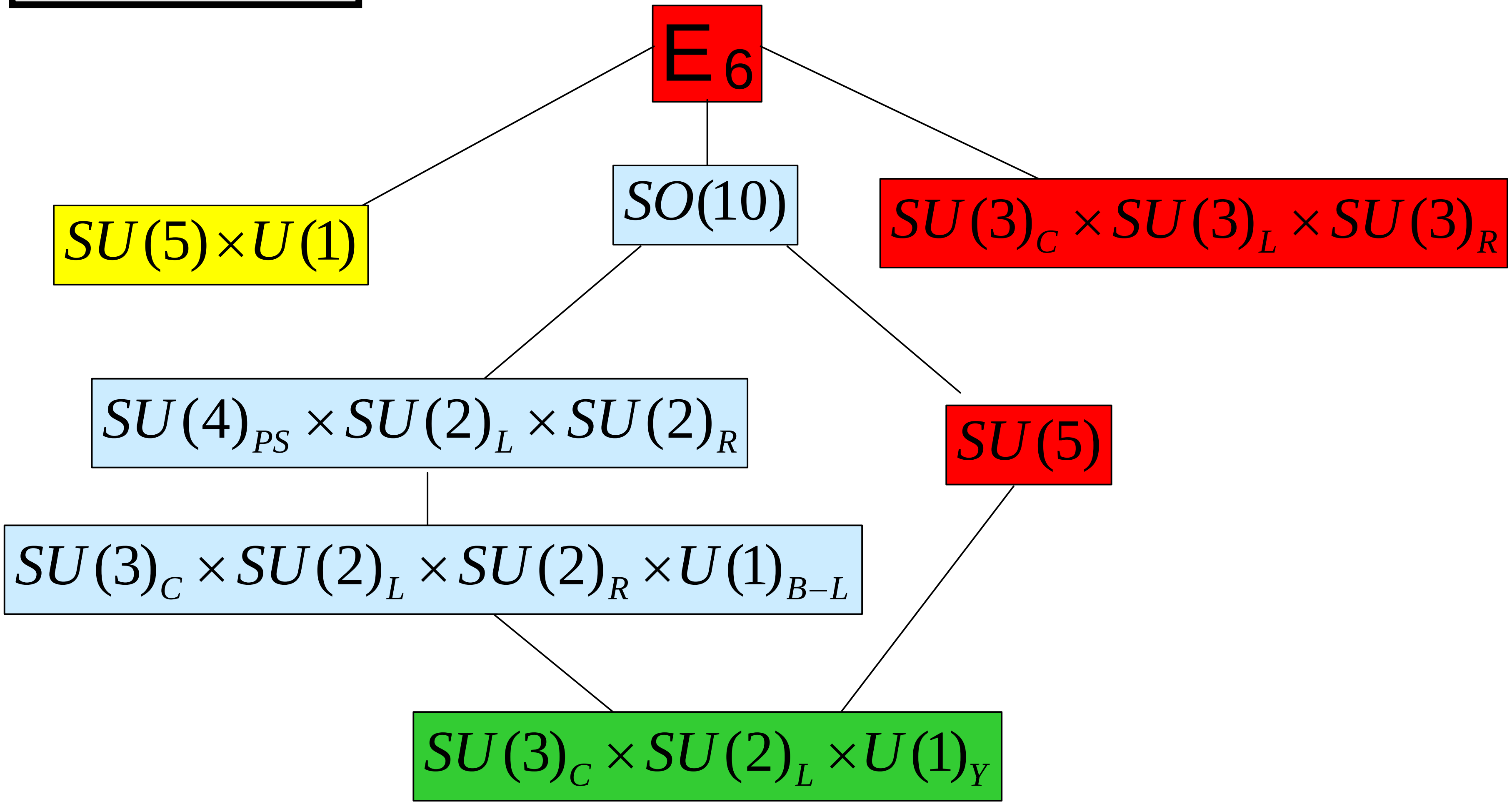
Family × GUT symmetry approach

Many models have been constructed:

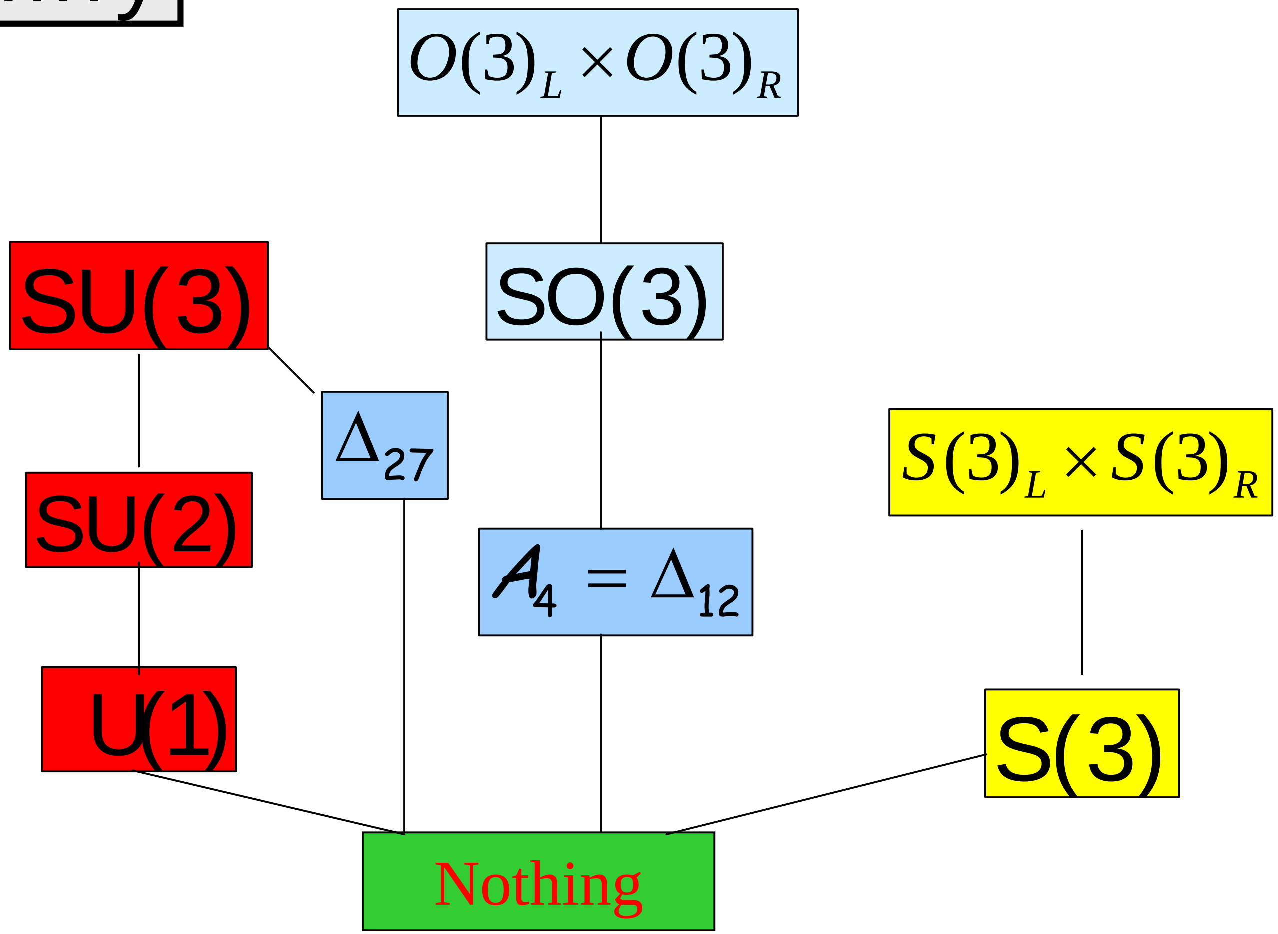
Babu, Mohapatra, Chen, Mahanthappa, Ma, Cheng,Everett, Ramond, Altarelli, Feruglio, King, Ross, Varzielas,Velasco-Sevilla, ….



G_{GUT}

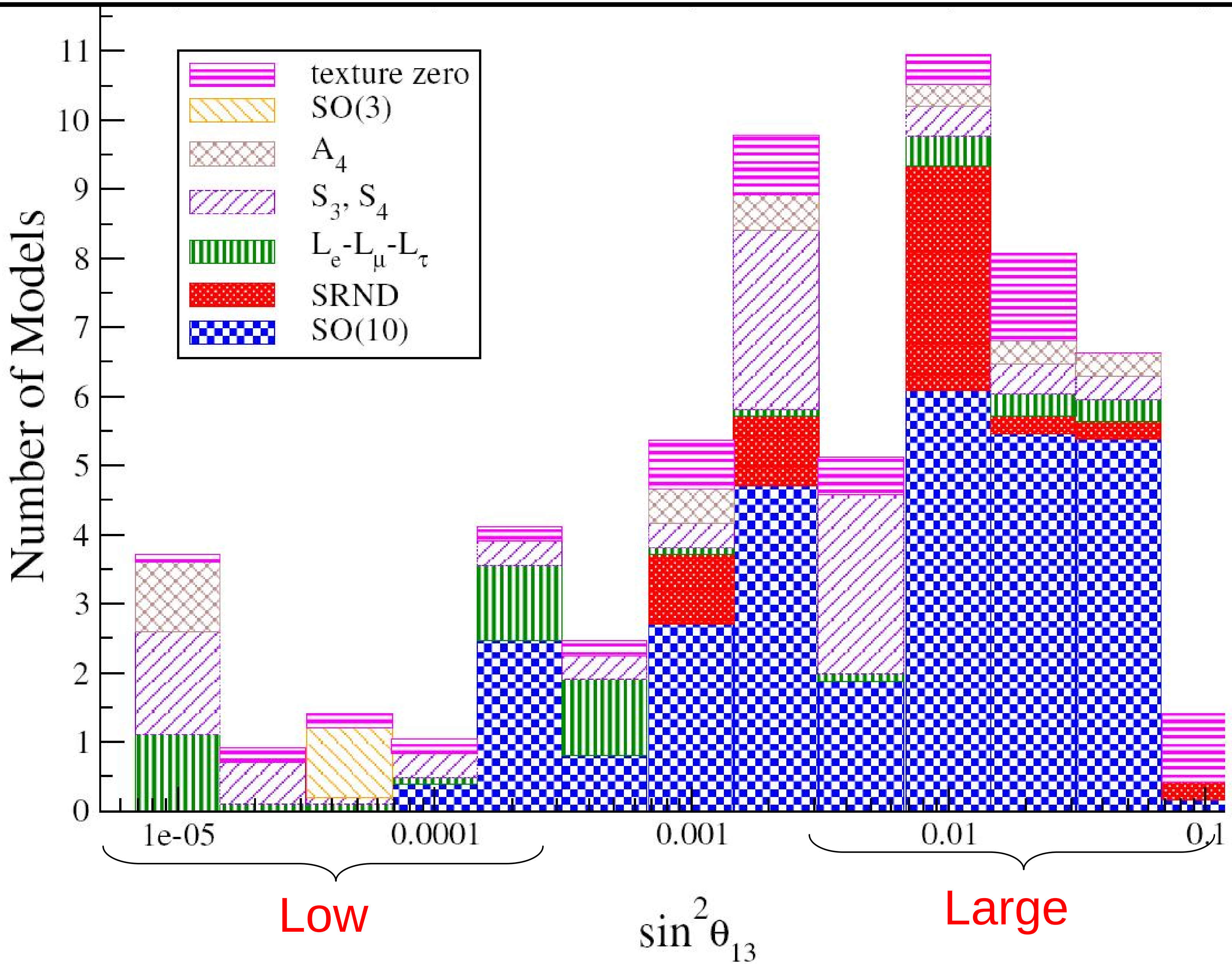

E_6
$SU(5)\times U(1)$
$SO(10)$
$SU(3)_C \times SU(3)_L \times SU(3)_R$
$SU(4)_{PS} \times SU(2)_L \times SU(2)_R$
$SU(5)$
$SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$
$SU(3)_C \times SU(2)_L \times U(1)_Y$

G_{Family}



$O(3)_L \times O(3)_R$
SO(3)
SU(3)
Δ_{27}
SU(2)
$A_4 = \Delta_{12}$
$S(3)_L \times S(3)_R$
U(1)
S(3)
Nothing

Model predictions for θ_{13} Albright and Chen



GUT relations

Y_u ⟷ V_{CKM} ⟷ Y_d

Class of models: $\theta_{12}^{d} \gg \theta_{12}^{u}$

$\theta_{12}^{d} \approx \theta_C$

See-saw ⇒
$m_\nu = v_{EW}^2 Y_\nu M_R^{-1} Y_\nu^T$

GUT relation

m_ν ⟷ V_{MNS} ⟷ Y_e

$$\theta_{12}^{e} \approx \frac{\theta_{12}^{d}}{3} \approx \frac{\theta_C}{3}$$

e.g. $$Y^u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & 2\varepsilon^2 & 2\varepsilon^2 \\ \varepsilon^3 & 2\varepsilon^2 & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} 0 & \bar\varepsilon^3 & \bar\varepsilon^3 \\ \bar\varepsilon^3 & \bar\varepsilon^2 & \bar\varepsilon^2 \\ \bar\varepsilon^3 & \bar\varepsilon^2 & 1 \end{pmatrix}, \quad Y^e = \begin{pmatrix} 0 & \bar\varepsilon^3 & \bar\varepsilon^3 \\ \bar\varepsilon^3 & 3\bar\varepsilon^2 & 3\bar\varepsilon^2 \\ \bar\varepsilon^3 & 3\bar\varepsilon^2 & 1 \end{pmatrix}. \quad \varepsilon \approx 0.05, \bar\varepsilon \approx 0.15$$

Roberts,Romanino,
Ross,Velasco-Sevilla

Predictions for θ_{12} and θ_{13}

Bjorken; Ferrandis, Pakvasa; SFK

$$U_{MNS} = V^{E_L} V^{\nu_L \dagger}$$

Cabibbo-like

Tri-bimaximal

$$\longrightarrow \quad \theta_{13} \approx \frac{\theta_{12}^e}{\sqrt{2}} \approx \frac{\theta_C}{3\sqrt{2}} \approx 3^\circ,$$

$$\longrightarrow \quad \theta_{12}^o - 35^o \approx \theta_{13}^o \cos\delta$$

Sum Rule SFK, Antusch,Masina

c.f. Cabibbo Haze analysis of Everett, Ramond with a ``left-Cabibbo shift''

$$\mathcal{U}_{\mathrm{PMNS}} = \mathcal{V}(\lambda)\,\mathcal{W} \qquad V(\lambda) \approx \begin{pmatrix} 1 & (\lambda/3)e^{-i\delta} & 0 \\ -(\lambda/3)e^{i\delta} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad W = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

In terms of the mixing matrix

$$U_{MNS} = V^{E_L} V^{\nu_L \dagger} \approx \begin{pmatrix} 1 & (\lambda/3)e^{-i\delta} & 0 \\ -(\lambda/3)e^{i\delta} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Cabibbo-like

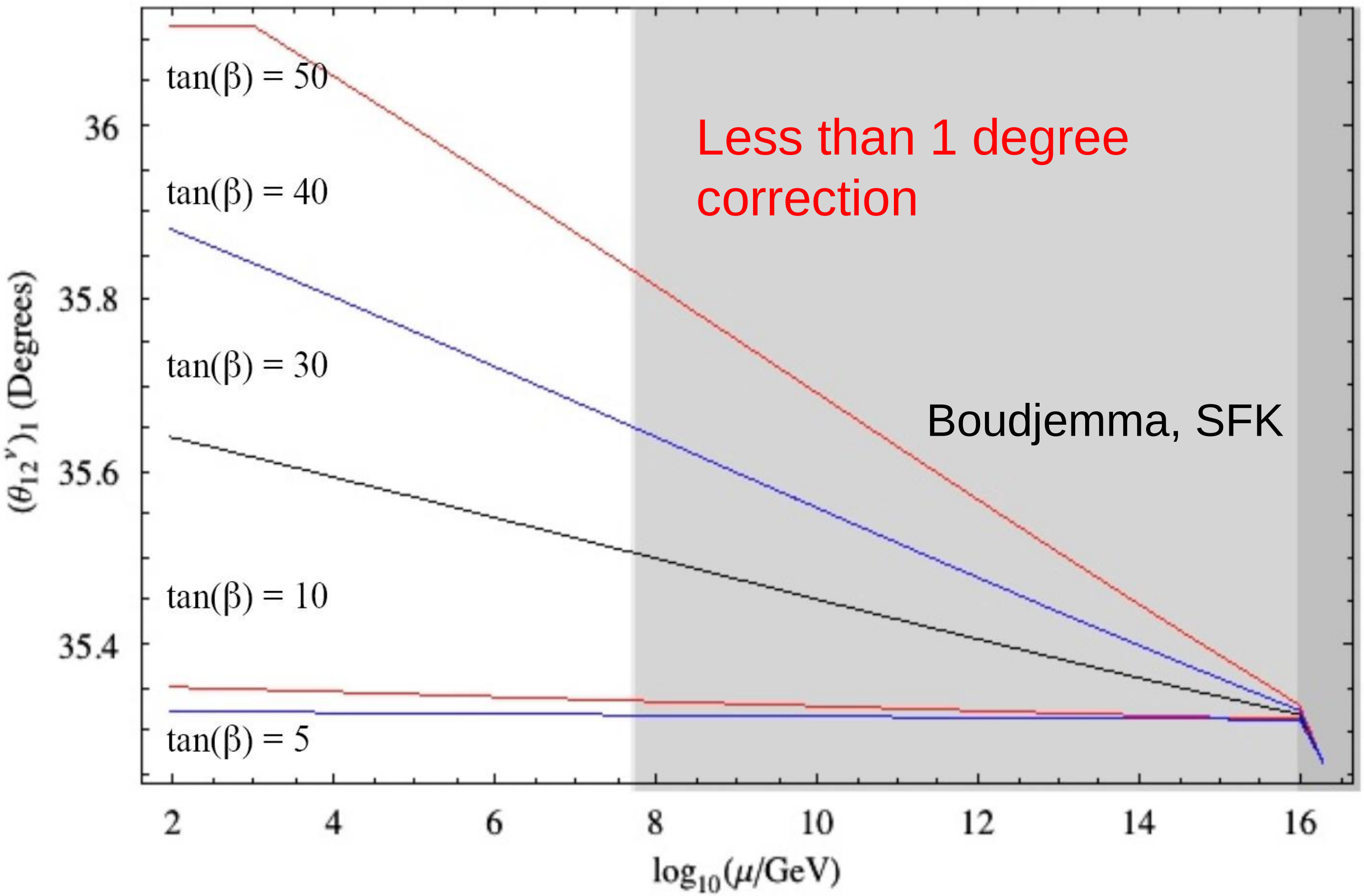
Tri-bimaximal

λ = Wolfenstein

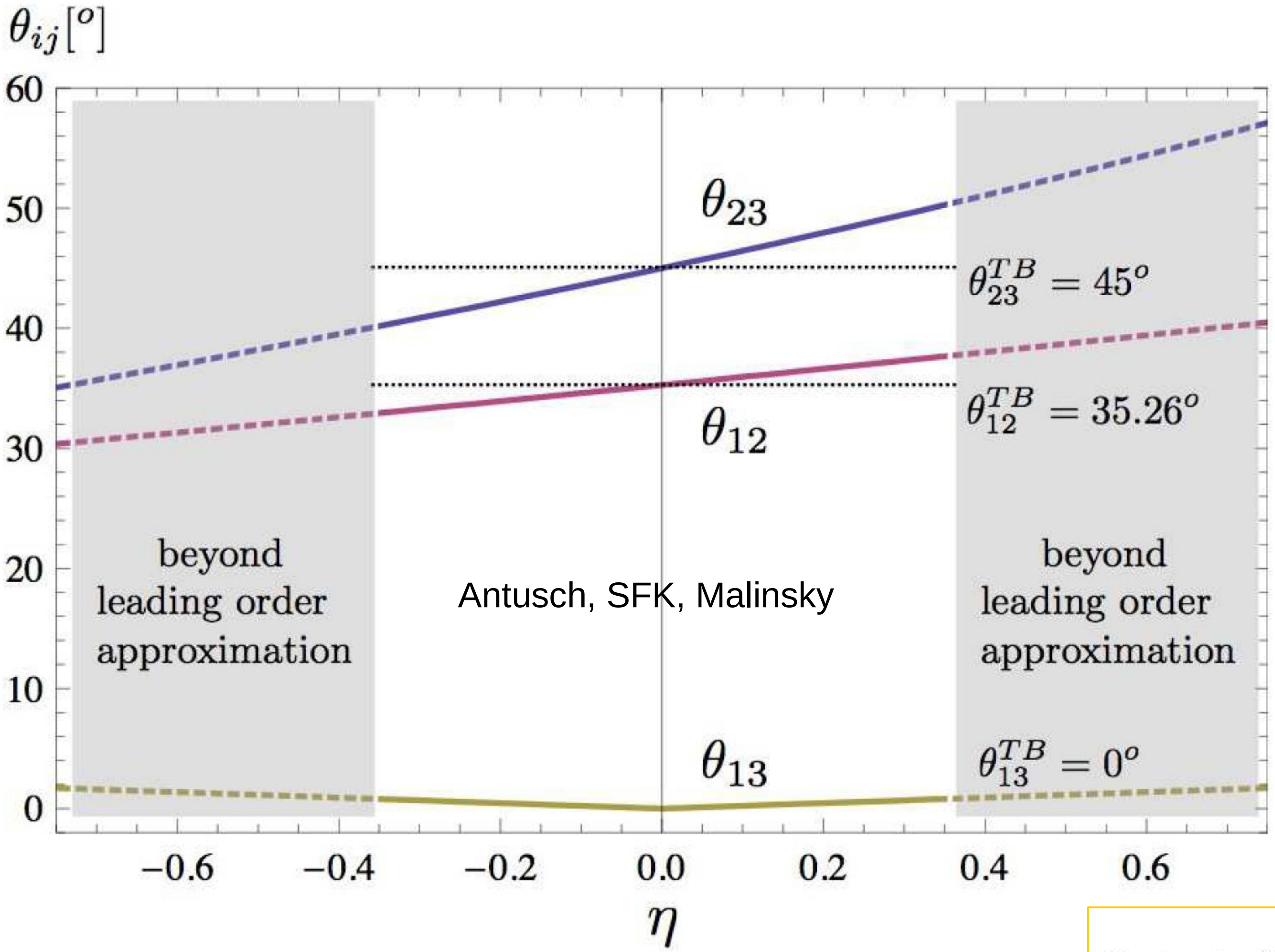
$$U_{MNS} \approx \begin{pmatrix} \sqrt{\frac{2}{3}}\left(1-(\lambda/6)\cos\delta\right) & \frac{1}{\sqrt{3}}\overbrace{\left(1+(\lambda/3)\cos\delta\right)}^{s} & \frac{1}{\sqrt{2}}\overbrace{(\lambda/3)}^{r}e^{-i\delta} \\ -\frac{1}{\sqrt{6}}\left(1+(2\lambda/3)\cos\delta\right) & \frac{1}{\sqrt{3}}\left(1-(\lambda/3)\cos\delta\right) & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

RGE corrections to sum rule

$$\theta_{12}^{o} \approx 35^{o} + \theta_{13}^{o} \cos\delta$$



Canonical/Kahler corrections to TBM



$$a = \frac{1}{4}\eta$$

$$s = \frac{1}{6}\eta$$

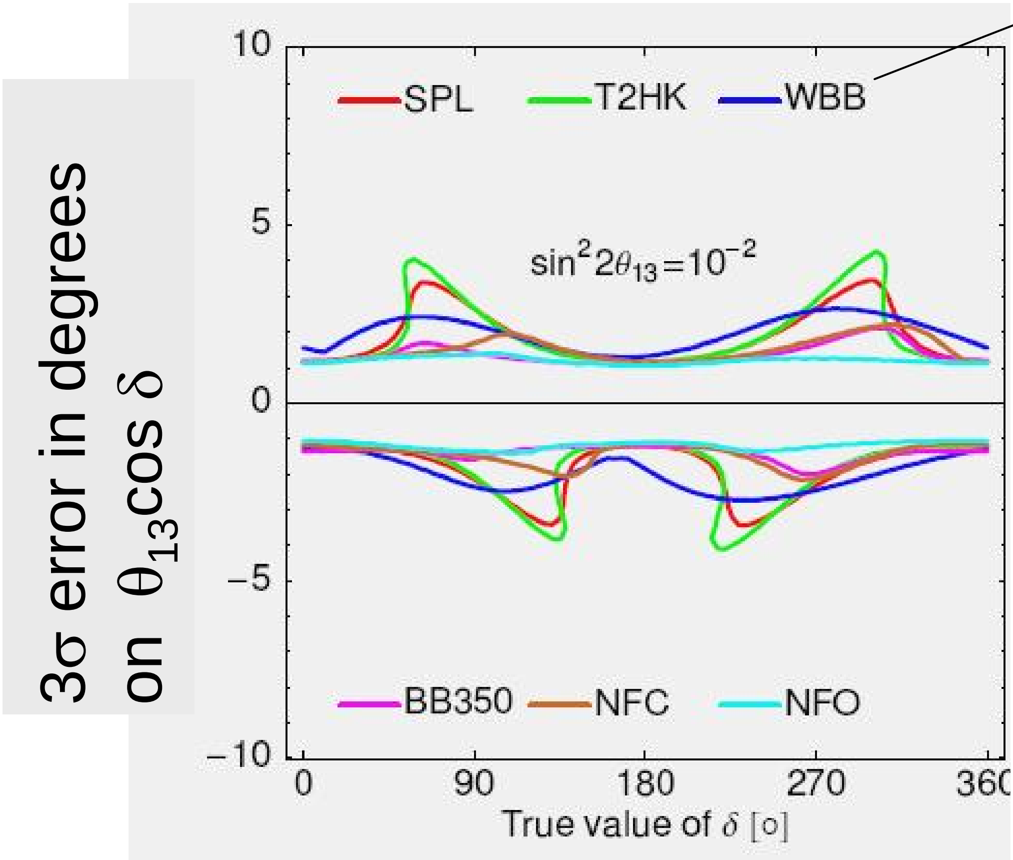
New sum rule

$$s = r\cos\delta + \frac{2}{3}a$$

η = third family wavefunction correction
(highly model dependent – could be small)

Experimental prospects to measure (r cos δ)

Project X



Antusch, Huber, SFK, Schwetz

LHC Implications – for SUSY

Ross, Vives, Velasco-Sevilla;
Antusch, SFK, Malinsky

Observation: SU(3) or Δ_{27} family symmetry predicts universal soft mass matrices in the symmetry limit

$$m_Q^2 \propto m_{u^c}^2 \propto m_{d^c}^2 \propto m_L^2 \propto m_{e^c}^2 \propto m_{N^c}^2 \propto \mathbb{1}$$

However Yukawa matrices and trilinear soft masses vanish in the SU(3) or Δ_{27} symmetry limit

In the real world where SU(3) or Δ_{27} is broken can perform an expansion in powers of small Yukawa coupling expansion parameters $\varepsilon \approx 0.05, \bar{\varepsilon} \approx 0.15$

If we impose CP symmetry spontaneously broken by flavon VEVs can also solve the SUSY CP Problem

Recall Yukawa matrices, ignoring phases:

$$Y^u = \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ \varepsilon^3 & 2\varepsilon^2 & 2\varepsilon^2 \\ \varepsilon^3 & 2\varepsilon^2 & 1 \end{pmatrix}, \quad Y^d = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & \bar{\varepsilon}^2 & 1 \end{pmatrix}, \quad Y^e = \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 3\bar{\varepsilon}^2 \\ \bar{\varepsilon}^3 & 3\bar{\varepsilon}^2 & 1 \end{pmatrix}.$$

Antusch, SFK, Malinsky

Under similar assumptions we predict at M_{GUT}:

$$m_Q^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix}, \quad m_{d^c}^2 \approx m_0^2 \begin{pmatrix} 1+\bar{\varepsilon}^4 & \bar{\varepsilon}^4 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^4 & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1+\mathcal{O}(1) \end{pmatrix}$$

$$m_{u^c}^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix},$$

Note strong third family non-universality

$$m_L^2 \approx m_0^2 \begin{pmatrix} 1+\varepsilon^4 & \varepsilon^4 & \varepsilon^4 \\ \varepsilon^4 & 1+\varepsilon^2 & \varepsilon^2 \\ \varepsilon^4 & \varepsilon^2 & 1+\mathcal{O}(1) \end{pmatrix}, \quad m_{e^c}^2 \approx m_0^2 \begin{pmatrix} 1+\bar{\varepsilon}^4 & \bar{\varepsilon}^4 & \bar{\varepsilon}^4 \\ \bar{\varepsilon}^4 & 1+\bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ \bar{\varepsilon}^4 & \bar{\varepsilon}^2 & 1+\mathcal{O}(1) \end{pmatrix}$$

Conclusion

- **Neutrino mass and mixing provides new insight into the flavour problem**
- **Precise TBM can be understood from the see-saw mechanism with sequential dominance**
- **This motivates a non-Abelian family symmetry**
- **GUTs plus family symmetry leads to quark-lepton relations leading to predictions for the reactor angle and testable sum rules**
- **Family symmetry can solve the SUSY flavour and CP problems and implies third family squark and slepton non-universality at the LHC**