Nonabelian Discrete Symmetries and the SUSY Flavor Problem

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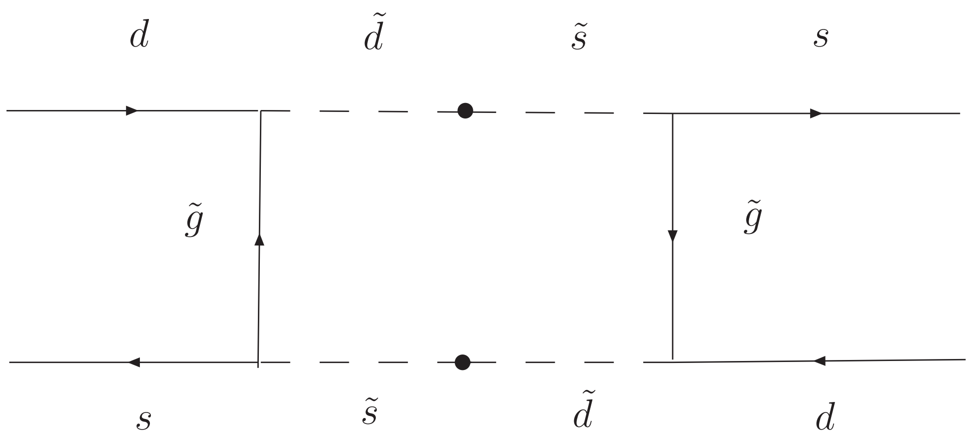
Outline

- SUSY Flavor Problem
- Motivation
- Model
- Summary and Conclusions

SUSY Flavor Problem

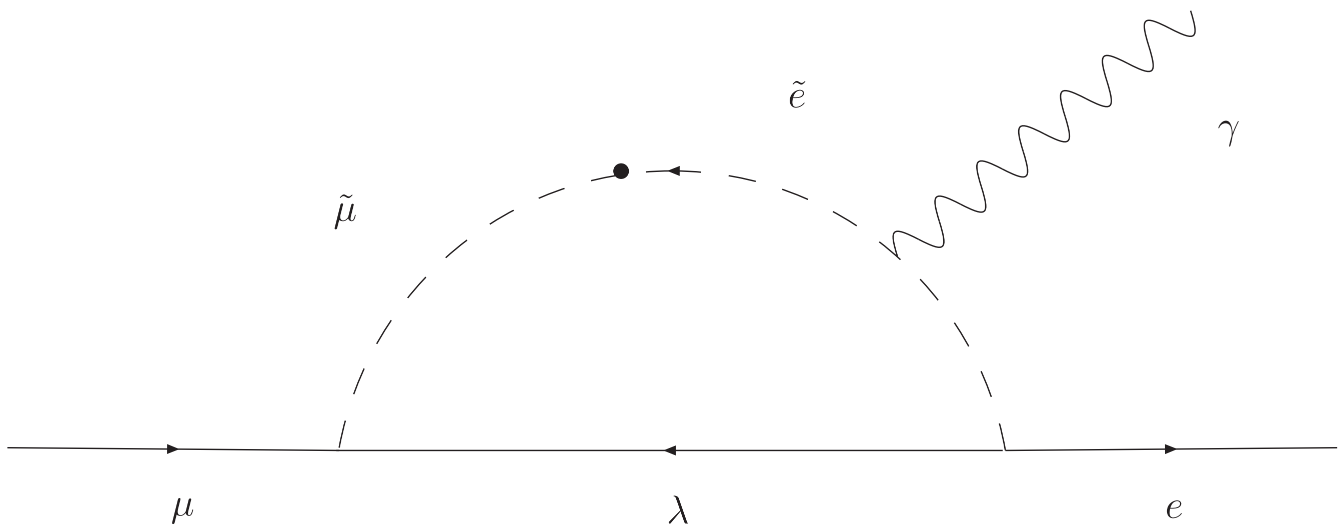
- Strongest constraint from $K^0 - \overline{K^0}$ mixing: $\Delta_{\tilde{d}\tilde{s}}/m_{sq}^2 < 10^{-4}$
- Solved by assuming universality:

$$m^2_{\tilde{d}\tilde{s}} = \begin{pmatrix} m_1^2 & 0 \\ 0 & m_1^2 \end{pmatrix}$$



- Constraint from $\mu \to e\gamma$: $\Delta_{\tilde{e}\tilde{\mu}}/m_{sl}^2 < 10^{-3}$
- It's similarly assumed:

$$m^2_{\tilde{e}\tilde{\mu}} = \begin{pmatrix} m_2^2 & 0 \\ 0 & m_2^2 \end{pmatrix}$$



Motivation

- It has been pointed out that the form

$$\begin{pmatrix} 0 & A & 0 \\ -A & 0 & B' \\ 0 & B & C \end{pmatrix}$$

for fermion mass matrices is consistent with phenomenology (Weinberg; Wilczek and Zee; Fritzsch)

- Factorizable Form (phases of A, B, B', C can be absorbed into fermion fields)

$\Longrightarrow$ We wish to obtain this with some symmetry

- Consider symmetry with 3 families belonging to $2 + 1$

• $(\tilde{d}, \tilde{s})$, $(\tilde{e}, \tilde{\mu})$ in same multiplets $\Longrightarrow$ Explains mass degeneracy

• To generate fermion masses use $2+1$ pairs of Higgs doublets: $(H_1^{u,d}, H_2^{u,d}) + H_3^{u,d}$

• For $2+1$ of $SU(2)$, fermion mass matrices have form

$$\begin{pmatrix} 0 & y_1 H_3 & y_2' H_2 \\ -y_1 H_3 & 0 & -y_2' H_1 \\ y_2 H_2 & -y_2 H_1 & y_3 H_3 \end{pmatrix}$$

• If $\langle H_1^u \rangle / \langle H_2^u \rangle = \langle H_1^d \rangle / \langle H_2^d \rangle$, 13 and 31 entries can be rotated away

- If full (local) $SU(2)$, D-terms cause FCNC problems $\Longrightarrow$ Discrete subgroups of $SU(2)$

- The Higgs mass matrix ($W = H_i^u M_{ij} H_j^d$)

$$M = \begin{pmatrix} 0 & a & cb_1 \\ -a & 0 & cb_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

can give large masses to all but one pair of doublets ("doublet-doublet splitting") $\Longrightarrow$ MSSM at low energy

$$H_0^u = \frac{b_2^* H_1^u - b_1^* H_2^u - a^* H_3^u}{\sqrt{|b_1|^2 + |b_2|^2 + |a|^2}}, H_0^d = \frac{c^* b_2^* H_1^d - c^* b_1^* H_2^d + a^* H_3^d}{\sqrt{|c|^2|b_1|^2 + |c|^2|b_2|^2 + |a|^2}}$$

Fermion Mass Matrices:

$$\frac{1}{\sqrt{|b_1|^2+|b_2|^2+|a|^2}}\begin{pmatrix} 0 & A_u a H_0^u & B'_u b_1 H_0^u \\ -A_u a H_0^u & 0 & B'_u b_2 H_0^u \\ B_u b_1 H_0^u & B_u b_2 H_0^u & C_u a H_0^u \end{pmatrix},$$

$$\frac{1}{\sqrt{|c|^2|b_1|^2+|c|^2|b_2|^2+|a|^2}}\begin{pmatrix} 0 & A_d a H_0^d & B'_d c b_1 H_0^d \\ -A_d a H_0^d & 0 & B'_d c b_2 H_0^d \\ B_d c b_1 H_0^d & B_d c b_2 H_0^d & C_d a H_0^d \end{pmatrix}$$

- 13,31 entries can be rotated away
- For real Yukawas, only complex c gives CP violation

The Group T'

- T' is the double covering of A_4

- Only subgroup of $SU(2)$ with doublets that are not self-conjugate

- Smallest subgroup of $SU(2)$ under which 3 does not break up

Representations of T':

- true singlet, 1
- conjugate pair of singlets, $1'$,$1''$
- real triplet, 3
- pseudoreal doublet, 2
- conjugate pair of doublets, $2'$,$2''$

$T' \times Z_6$ Model

$SU(2)_L$ **Doublets:**

$$H^u, H^d : (2', \omega);\ \ H_3^u, H_3^d : (1', \omega);\ \ H'^u, H'^d : (2, \omega^2);$$

$$H_3'^u, H_3'^d : (1', -\omega^2);\ \ H_3''^u : (1'', -\omega);\ \ H_3''^d : (1'', -\omega^2);$$

$$Q : (2', \omega);\ \ Q_3 : (1', \omega)$$

$SU(2)_L$ **Singlets:**

$$T : (3, 1);\ \ D : (2', -1);\ \ D' : (2'', -1);\ \ S_1 : (1, \omega^2);$$

$$S_2 : (1, \omega);\ \ S_3 : (1, -\omega);\ \ S_4 : (1, -\omega^2);\ \ S_5 : (1, -1);$$

$$Q^c : (2', \omega);\ \ Q_3^c : (1', \omega)$$

- $\omega = e^{i\frac{2\pi}{3}}$
- Assignment commutes with $SO(10)$ Grand Unification

Superpotential for SM Singlet Higgs:

$$W = a_1 DD' + a_2 T^2 + b_1 T^3 + b_2 D^2 T + b_3 D'^2 T + b_4 DD'T$$
$$+ a_3 S_1 S_2 + a_4 S_3 S_4 + a_5 S_5^2 + b_5 S_1^3 + b_6 S_2^3 + b_7 S_2 S_3^2$$
$$+ b_8 S_1 S_4^2 + b_9 S_1 S_3 S_5 + b_{10} S_2 S_4 S_5$$

- Can generate VEV's for all fields
- No flat directions or accidental symmetries

Higgs Doublet Mass Matrix:

$$\begin{pmatrix} 0 & 0 & 0 & \beta(T_1 - i\omega^2 T_2) & -\beta\omega T_3 & \delta D_2 & 0 \\ 0 & 0 & 0 & -\beta\omega T_3 & -\beta(T_1 + i\omega^2 T_2) & -\delta D_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \zeta S_3 \\ \alpha(T_1 - i\omega^2 T_2) & -\alpha\omega T_3 & 0 & 0 & \lambda S_1 & 0 & 0 \\ -\alpha\omega T_3 & -\alpha(T_1 + i\omega^2 T_2) & 0 & -\lambda S_1 & 0 & 0 & 0 \\ \gamma D_2 & -\gamma D_1 & 0 & 0 & 0 & 0 & \xi S_2 \\ 0 & 0 & \epsilon S_2 & 0 & 0 & m & 0 \end{pmatrix}$$

Integrating out $H'^{u,d}, H_3'^{u,d}, H_3''^{u,d}$:

$$\begin{pmatrix} 0 & a & cb_1 \\ -a & 0 & cb_2 \\ b_1 & b_2 & 0 \end{pmatrix}$$

$$a = \frac{\alpha\beta}{\lambda}\frac{\langle T^2\rangle}{\langle S_1\rangle},\ b_1 = \frac{\gamma\zeta}{\xi}\frac{\langle S_3 D_2\rangle}{\langle S_2\rangle},\ b_2 = -\frac{\gamma\zeta}{\xi}\frac{\langle S_3 D_1\rangle}{\langle S_2\rangle},\ c = \frac{\delta\epsilon\xi}{\gamma\zeta}\frac{\langle S_2^2\rangle}{\langle S_3\rangle}$$

- Light modes couple to SM singlet Higgs, $H_0^u H_0^d \Phi$, with $\langle\Phi\rangle = 0$ in SUSY limit
- After SUSY breaking $\langle\Phi\rangle \sim M_{SUSY}$

- c complex $\Longrightarrow$ spontaneous CP violation
- Complex B_μ-parameter generated $\Longrightarrow$ SUSY CP problem not fully solved

- Discrete symmetries should come from broken local symmetries so that they are respected by gravity
- When $SU(2)$ reps. break up under T', $1', 1''$ and $2', 2''$ always occur in pairs $\Longrightarrow$ This model can be difficult to obtain from a local symmetry (e.g. $4 \rightarrow 2' + 2''$, $5 \rightarrow 3 + 1' + 1''$, $6 \rightarrow 2 + 2' + 2''$)
- By extending the Abelian part of the symmetry, the model can be altered to use only complete multiplets of $SU(2)$

$T' \times Z_3 \times Z_6$ Model

$SU(2)_L$ **Doublets:**

$$H^u, H^d : (2, \omega, \omega);\; H_3^u, H_3^d : (1, \omega, \omega);\; H'^u, H'^d : (2, 1, \omega^2);$$

$$H_3'^u, H_3'^d : (1, \omega, -\omega^2);\; H_3''^u : (1, \omega^2, -\omega);\; H_3''^d : (1, \omega^2, -\omega^2);$$

$$Q : (2, \omega, \omega);\; Q_3 : (1, \omega, \omega)$$

$SU(2)_L$ **Singlets:**

$$T : (3, \omega, 1);\; T' : (3, \omega^2, 1);\; D : (2, \omega, -1);\; D' : (2, \omega^2, -1);$$

$$S_1 : (1, 1, \omega^2);\; S_2 : (1, 1, \omega);\; S_3 : (1, 1, -\omega);\; S_4 : (1, 1, -\omega^2);$$

$$S_5 : (1, 1, -1);\; Q^c : (2, \omega, \omega);\; Q_3^c : (1, \omega, \omega)$$

- $SU(2)$ can be broken to T' with a 7

Summary

A Model with the following properties:

- MSSM at low energy
- Solves SUSY flavor problem
- Ameliorates SUSY CP problem
- Solves μ problem
- Consistent with Grand Unification