Interaction of Dirac & Majorana Neutrinos with Weak Gravitational Fields

(Arun M Thalapillil, Univ. of Chicago)

- Neutrinos are produced copiously in many highly energetic astrophysical processes. (Solar, SN, GZK, ...)

$$E_\nu \sim \text{MeV} \rightarrow 10^{12} \text{ GeV}$$

- Neutrinos being very weakly interacting travel almost unhindered to earth. Many planned and ongoing neutrino experiments aim to detect these UHE neutrinos. (ANITA, ICECUBE, ...)
- Neutrinos only interact through electroweak and gravitational interactions.
- It is our aim to study how these UHE *neutrinos interact with weak gravitational fields*. (A. Menon and A.M.T ; hep-ph/0804.3833v1)

K. Greisen (1966)
G.T. Zatsepin and V.A. Kuzmin (1966)
J. Bahcall (1989)

Weak Gravitational Fields

- A gravitational field may be considered weak when

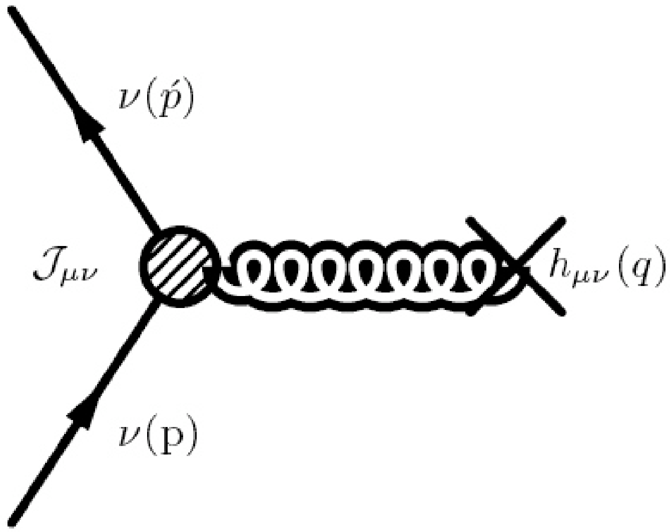
$$g_{\mu\nu} \simeq \eta_{\mu\nu} + \kappa h_{\mu\nu} + \mathcal{O}(h^2) \qquad \delta_g \sim |\kappa h^{\mu\nu}| \ll 1$$

- Assuming spherical symmetry, some typical examples have

$$\begin{aligned} \delta_{\odot} &\approx \ 10^{-6} \\ \delta_{AGN} &\approx \ 10^{-5} \\ \delta_{N} &\approx \ 0.3 \end{aligned}$$

- The interaction term we are interested in is

$$\kappa \ h^{\mu\nu} \ \bar{u}(p^{'})_i i \mathcal{J}_{\mu\nu} u(p)_i$$



The Graviton-Neutrino Vertex

- Using Lorentz invariance (rank 2 symmetric tensor) and gauge invariance (Slavnov-Taylor-Ward identities) we have the mass diagonal tensor current

$$i\mathcal{J}^{'}_{\mu\nu}[i,i] = \Big[F_1(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu) + F_2(q^2)(r_\mu r_\nu) + F_3(q^2)(\gamma_{\{\mu} r_{\nu\}})\Big]$$

$$q = p^{'} - p \quad , \quad r = p^{'} + p$$

- If we include parity violating operators then we have 3 more mass diagonal form factors

$$i\mathcal{J}^{''}_{\mu\nu}[i,i] = \Big[G_1(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma^5 + G_2(q^2)(r_\mu r_\nu)\gamma^5 + G_3(q^2)\{q^2(\gamma_{\{\mu} r_{\nu\}}) - 2m_{\nu_i}(q_{\{\mu} r_{\nu\}})\}\gamma^5\Big]$$

- For the mass off-diagonal case, again by Lorentz and gauge invariance

$$\bar{u}(p^{'})_j i\mathcal{J}^{'}_{\mu\nu}\,[i,j]\,u(p)_i = \bar{u}(p^{'})_j \Big[E_1(q^2)(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

$$+E_2(q^2)(r_\mu r_\nu) + E_3(q^2)\{q^2(\gamma_{\{\mu} r_{\nu\}}) - \Delta_{ij} m_\nu (q_{\{\mu} r_{\nu\}})\}\Big] u(p)_i$$

- Similarly, the mass off-diagonal parity violating operators are

$$q = p^{'} - p \quad , \quad r = p^{'} + p$$

$$\bar{u}(p^{'})_j i\mathcal{J}^{''}_{\mu\nu}\,[i,j]\,u(p)_i = \bar{u}(p^{'})_j \Big[D_1(q^2)\gamma^5(q^2 g_{\mu\nu} - q_\mu q_\nu)$$

$$+D_2(q^2)(r_\mu r_\nu)\gamma^5 + D_3(q^2)\{q^2(\gamma_{\{\mu} r_{\nu\}}) - \Sigma_{ij} m_\nu (q_{\{\mu} r_{\nu\}})\}\gamma^5\Big] u(p)_i$$

Non Relativistic Interpretations of the Form Factors.

- Consider

$$\lim_{q\to 0} \mathcal{H}_g = \kappa h^{\mu\nu}\ \bar{u}(p)_i i\mathcal{J}_{\mu\nu} u(p)_i$$

$$\kappa h^{00}\bar{u}(0)_i i\mathcal{J}_{00} u(0)_i \approx m_{\nu_i}\Phi_{\rm g}\left[F_2(0)+F_3(0)\right]\phi^\dagger\phi$$

- To satisfy the weak equivalence principle we require

$$F_2(0) +\ F_3(0) =\ 1$$

- From angular momentum conservation we also have

$$F_2(0) \to 0$$

- This leads to the definition of a neutrino mass radius

$$F_3(q^2) \sim \int \rho^{\nu}_{\rm m}(r) e^{i\vec{q}\cdot\vec{r}} d^3r$$

$$\langle r^2_\nu \rangle_m \sim -6 \frac{d}{dq^2} \left[F_3(q^2)\right] \Big|_{q^2 \to 0}$$

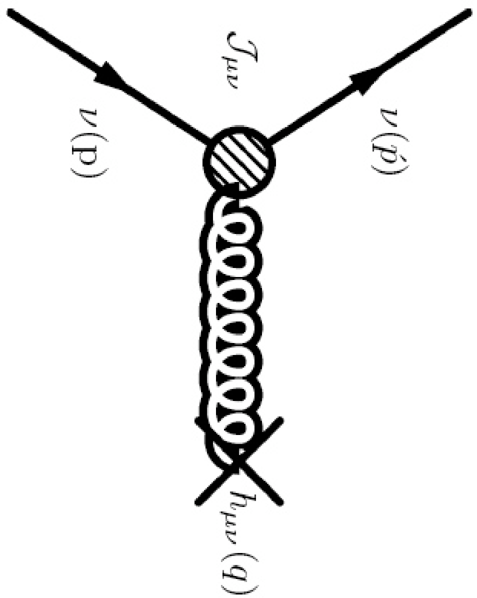
- From the virial theorem, the incoming neutrino energy density is

$$\langle \rho^{\nu,h}_E \rangle_{\rm t} \approx \left\langle m_\nu \left[F_2(q^2) + F_3(q^2)\right] \phi^\dagger \phi \right\rangle_{\rm t} + \left\langle q^2 \left[4F_1(q^2) - \frac{F_2(q^2)}{4m_\nu}\right] \phi^\dagger \phi \right\rangle_{\rm t}$$

Vertex Symmetries and Angular Momentum States.

- Consider the s-channel process with identical Majorana neutrinos. Then from Fermi-Dirac Statistics we have the constraint

$$\left[\hat{\mathcal{C}}^{-1}\mathcal{J}^{M}_{\mu\nu}\left[(p^{'},s^{'}),(p,s)\right]\hat{\mathcal{C}}\right]^{T}=\mathcal{J}^{M}_{\mu\nu}\left[(p,s),(p^{'},s^{'})\right]$$



- This implies that, by spin-statistics

$$\boxed{G^{M}_{3}(q^{2}) \;=\; 0}$$

Neutrino-Photon vertex : B. Kayser (1982);
B. Kayser and A. S. Goldhaber (1983).
A. Khare and J. Oliensis (1984)
K. L. Ng (1993)

- Hence a Majorana neutrino only has 5 gravitational form factors compared to 6 gravitational form factors for a Dirac neutrino.

- For the mass off-diagonal case, crossing symmetry implies that all the processes below have the same form factors for a Majorana neutrino

$$\nu_i^M \xrightarrow{h^*} \nu_j^M \;\Leftrightarrow\; h^* \longrightarrow \nu_i^M + \nu_j^M \;\Leftrightarrow\; \nu_j^M \xrightarrow{h^*} \nu_i^M$$

- An off-shell graviton has a J=0 as well as a J=2 component. Hence the possible final angular momentum states (for Dirac & Majorana) are

D. Dicus and S. Willenbrock (2005)
H. van Dam and M. Veltman (1970)

$$J = 0 \;:\quad S = 0 \;:\quad {}^1S_0$$
$$S = 1 \;:\quad {}^3P_0$$

$$J = 2 \;:\quad S = 0 \;:\quad {}^1D_2$$
$$S = 1 \;:\quad {}^3P_2,\ {}^3D_2,\ {}^3F_2$$

- Under the permutation operator

$$\mathbb{P}_{12}^{M}\left({}^{2S+1}L_J\right) \;=\; (-1)^{L+S+1}$$

- For identical Majorana neutrinos in the final state all states are anti-symmetric except

$$\mathbb{P}_{12}^{M}\left({}^{3}D_2\right) \;=\; +1$$

- The angular momentum eigenfunctions may be derived to be

$$
\begin{aligned}
{}^1S_0 &: \quad S \\
{}^3P_0 &: \quad \hat{p} \cdot \vec{T} \\
{}^1D_2 &: \quad (3\,\hat{p} \otimes \hat{p} - 1 \otimes 1)\;\; S \\
{}^3P_2 &: \quad \hat{p} \otimes \vec{T},\;\; (\hat{p} \cdot \vec{T})\; 1 \otimes 1 \\
{}^3D_2 &: \quad \hat{p} \;\otimes\; (\vec{T} \times \hat{p}) \\
{}^3F_2 &: \quad 5\;(\hat{p} \cdot \vec{T})\; \hat{p} \otimes \hat{p} + (\hat{p} \cdot \vec{T})\; 1 \otimes 1 - \hat{p}
\end{aligned}
$$

$$\phi_2^\dagger \phi_1^c = S \quad \phi_2^\dagger \vec{\sigma} \phi_1^c = \vec{T}$$

- The graviton polarization tensor may be constructed from the spin-1 polarization vectors

$$\epsilon_{\mu\nu}[J, J_3] = \sum_{j_3 + j_3^{'} = J_3} \mathcal{C}(J\,1\,1; J_3\, j_3\, j_3^{'})\, \eta_\mu[1, j_3]\, \eta_\nu[1, j_3^{'}]$$

- Taking the non-relativistic limit and using the properties of the graviton polarization tensor gives

$$\epsilon^{\mu\nu}[0,J_3]\cdot\bar{u}_2\Big(q^2 g_{\mu\nu}-q_\mu q_\nu\Big)v_1 \xrightarrow{NR} -\frac{\sqrt{3}q^2}{m_\nu}\hat{p}\cdot\vec{T}$$

$$\epsilon^{\mu\nu}[2,J_3]\cdot\bar{u}_2\Big(\gamma_\mu r_\nu+\gamma_\nu r_\mu\Big)v_1 \xrightarrow{NR} \frac{2\epsilon^{ij}}{5m_\nu^2}\Big[5(\hat{p}\cdot\vec{T})\hat{p}_i\hat{p}_j$$
$$+(\hat{p}\cdot\vec{T})\delta_{ij}-(\hat{p}_{\{i}\vec{T}_{j\}})\Big]-\frac{\epsilon^{ij}}{10m_\nu^2}\Big[4(\hat{p}\cdot\vec{T})\delta_{ij}+(\hat{p}_{\{i}\vec{T}_{j\}})\Big]$$

$$\epsilon^{\mu\nu}[0,J_3]\cdot\bar{u}_2\Big(\gamma_\mu r_\nu+\gamma_\nu r_\mu\Big)v_1 \xrightarrow{NR} \frac{\hat{p}\cdot\vec{T}}{m_\nu^2}$$

$$\epsilon^{\mu\nu}[2,J_3]\cdot\bar{u}_2(r_\mu r_\nu)v_1 \xrightarrow{NR} \frac{4\epsilon^{ij}}{5m_\nu}\Big[5(\hat{p}\cdot\vec{T})\hat{p}_i\hat{p}_j$$
$$+(\hat{p}\cdot\vec{T})\delta_{ij}-(\hat{p}_{\{i}\vec{T}_{j\}})\Big]+\frac{4\epsilon^{ij}}{5m_\nu}\Big[(\hat{p}_{\{i}\vec{T}_{j\}})-(\hat{p}\cdot\vec{T})\delta_{ij}\Big]$$

$$\epsilon^{\mu\nu}[0,J_3]\cdot\bar{u}_2(r_\mu r_\nu)v_1 \xrightarrow{NR} \frac{4\hat{p}\cdot\vec{T}}{m_\nu}$$

- Similarly for the parity violating part, in the non-relativistic limit we have

$$\epsilon^{\mu\nu}[0,J_3]\cdot\bar{u}_2\left(q^2 g_{\mu\nu}-q_\mu q_\nu\right)\gamma^5 v_1 \xrightarrow{NR} \sqrt{3}q^2\left(1+\frac{p^2}{4m_\nu^2}\right)S$$

$$\epsilon^{\mu\nu}[2,J_3]\cdot\bar{u}_2\left(\gamma_\mu r_\nu+\gamma_\nu r_\mu\right)\gamma^5 v_1 \xrightarrow{NR} \frac{2\epsilon^{ij}}{m_\nu}\left[\hat{p}_{\{i}(\hat{p}\times\vec{T})_{j\}}\right]$$

$$\epsilon^{\mu\nu}[2,J_3]\cdot\bar{u}_2(r_\mu r_\nu)\gamma^5 v_1 \xrightarrow{NR} -\frac{4}{3}\left(1+\frac{p^2}{4m_\nu^2}\right)\epsilon^{ij}\left(3\hat{p}_i\hat{p}_j-\delta_{ij}\right)S$$

$$\epsilon^{\mu\nu}[0,J_3]\cdot\bar{u}_2(r_\mu r_\nu)\gamma^5 v_1 \xrightarrow{NR} 4\left(1+\frac{p^2}{4m_\nu^2}\right)S$$

- We have then have the angular momentum associations

$$
\begin{aligned}
q^2 g_{\mu\nu} - q_\mu q_\nu &: \quad {}^3P_0 \\
(q^2 g_{\mu\nu} - q_\mu q_\nu)\gamma_5 &: \quad {}^1S_0 \\
\gamma_\mu r_\nu + \gamma_\nu r_\mu &: \quad {}^3F_2 \oplus {}^3P_2 \; ; \; {}^3P_0 \\
(\gamma_\mu r_\nu + \gamma_\nu r_\mu)\gamma_5 &: \quad {}^3D_2 \\
r_\mu r_\nu &: \quad {}^3F_2 \oplus {}^3P_2 \; ; \; {}^3P_0 \\
(r_\mu r_\nu)\gamma_5 &: \quad {}^1D_2 \; ; \; {}^1S_0
\end{aligned}
$$

- Let us consider the constraints from CP invariance

$$\zeta^{\mathrm{D}}_{\mathrm{CP}}\left({}^{2S+1}L_J\right) \;=\; (-1)^{S+1}$$

$$\zeta^{\mathrm{M}}_{\mathrm{CP}}\left({}^{2S+1}L_J\right) \;=\; (-1)^{L+1}$$

$$\boxed{G_1^D(q^2) \xrightarrow{CP} 0 \qquad G_2^D(q^2) \xrightarrow{CP} 0}$$

$$\boxed{G_1^M(q^2) \xrightarrow{CP} 0 \qquad G_2^M(q^2) \xrightarrow{CP} 0}$$

- For the off-diagonal case, for Majorana neutrinos

$$\zeta^{\mathrm{M}}_{\mathrm{CP}}\left({}^{2S+1}L_J\right) = \eta_i^*\eta_j^*(-1)^L$$

- In the original t-channel we then have

$$\boxed{\eta_i^*\eta_j^* = -1 \xrightarrow{CP} E_1(q^2) = E_2(q^2) = E_3(q^2) = 0}$$

$$\boxed{\eta_i^*\eta_j^* = +1 \xrightarrow{CP} D_1(q^2) = D_2(q^2) = D_3(q^2) = 0}$$

- Practical Majorana-Dirac Confusion Theorems may be shown to be still valid in neutrino-graviton interactions just as in the neutrino electromagnetic interaction case

B. Kayser (1982)

$$\frac{F_3^M(q^2)}{4} \simeq \frac{F_3^D(q^2)}{4} + q^2 G_3^D(q^2)$$

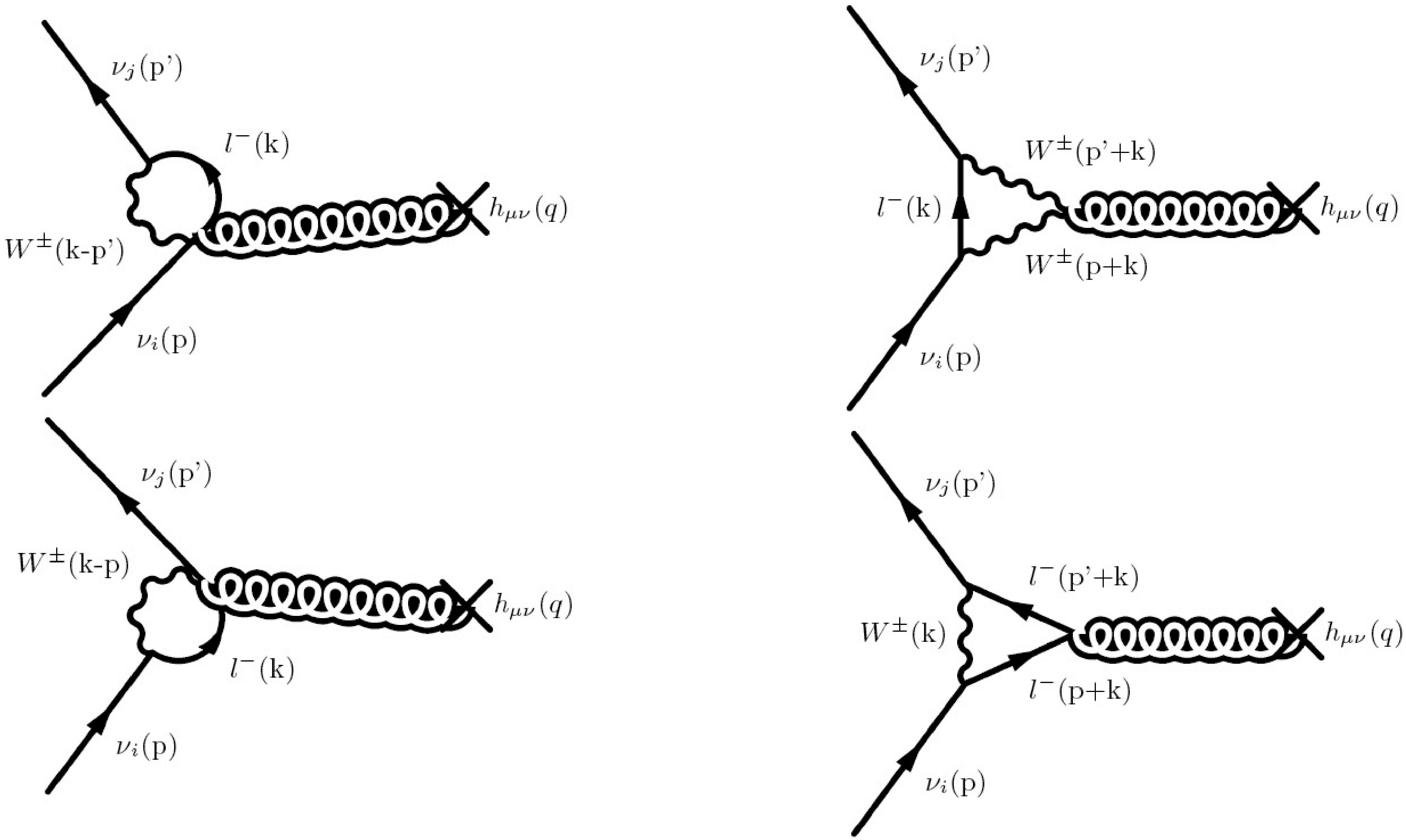
- This renders any difference between Majorana and Dirac neutrinos in gravitational interactions ineffective.

- Therefore our conclusion disagrees with some recent claims in the literature.

D. Singh, N. Mobed and G. Papini (2006)

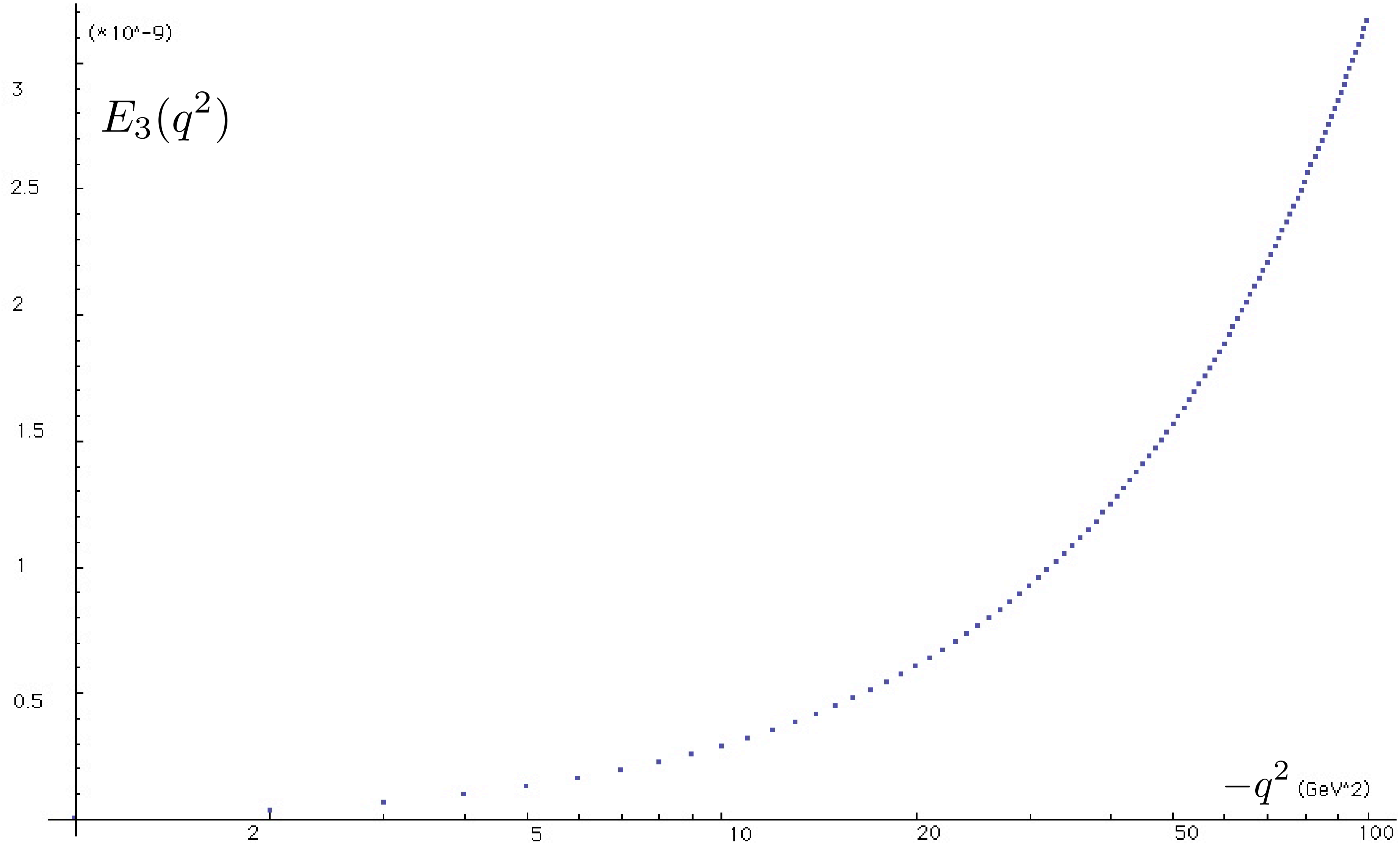
Neutrino Graviton Coupling at 1-Loop.

- Let us consider the neutrino transition form factors in gravity. The relevant Feynman diagrams are as shown



(Pheno 08, Arun M Thalapillil, Univ. of Chicago)

- The graviton-neutrino transition form factor has a very weak dependence on the momentum transfer



(Pheno 08, Arun M Thalapillil, Univ. of Chicago)

- From the graviton-neutrino transition form factors one may calculate a neutrino transition mass radius

$$\langle r_\nu^2 \rangle_m [i \to j] \sim -6 \frac{d}{dq^2} \left[E_3(q^2) \right] \Big|_{q^2 \to 0}$$

- This gives, from our 1-loop calculations the transition mass radius

$$\langle r_\nu^2 \rangle_{\rm mass} [2 \to 3] \;\sim\; 1.5 \times 10^{-37} \text{ cm}^2$$

- For comparison, the transition charge radius calculation for a neutrino gives

$$\langle r_\nu^2 \rangle_{\rm charge} [2 \to 3] \;\sim\; 1.1 \times 10^{-33} \text{ cm}^2$$

- There is an intuitive way to understand the difference in order of magnitudes in terms of the Compton wavelengths.

$$\left\langle r_\nu^2 \right\rangle^\lambda_{\mathrm{charge}} \sim \frac{|q_{l^-}|\lambda_{l^-}^2 + |q_{W^+}|\lambda_{W^+}^2}{|q_{l^-}| + |q_{W^+}|}$$

$$\left\langle r_\nu^2 \right\rangle^\lambda_{\mathrm{mass}} \sim \frac{m_l \lambda_{l^-}^2 + M_W \lambda_{W^+}^2}{m_l + M_W}$$

- Dividing the two gives

$$\frac{\left\langle r_\nu^2 \right\rangle^\lambda_{\mathrm{mass}}}{\left\langle r_\nu^2 \right\rangle^\lambda_{\mathrm{charge}}} \approx 10^{-3}$$

Summary.

- UHE neutrinos have the potential to provide deep insights in particle physics and cosmology.
- In the interaction of neutrinos with weak gravitational fields, there are specific differences between Majorana and Dirac neutrinos.
- There is a connection between CP phases and vanishing of some of the gravitational neutrino form factors.
- The Practical Majorana-Dirac Confusion theorems are still valid in neutrino graviton interactions.
- The gravitational neutrino transition form factor has a weak dependence on momentum transfer and the neutrino transition mass radius is found to be smaller than the neutrino charge radius.