

# Small Neutrino Masses in a TeV Scale Seesaw Model with a $Z'$

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# Introduction

- Conventional Seesaw mechanism: High (GUT) scale physics
- With SM particle content:

$$C \frac{HH\bar{\ell}_L^c \ell_L}{\Lambda} \quad \text{small } \nu \text{ masses} \Rightarrow \Lambda \sim 10^{14} \text{ GeV} \quad \text{with} \quad C \sim \mathcal{O}(1)$$

- $\exists$  gauge singlet fermions  $\nu_R$

$$\begin{array}{l}
 H\bar{\ell}_L \nu_R \rightarrow m_D \\
 M_R \bar{\nu}_R^c \nu_R
 \end{array}
 \quad \text{integrating out } \nu_R : \quad M_{eff} \sim M_D M_R^{-1} M_D^T$$

$$y^2 \frac{HH\ell\ell}{M_R} \Rightarrow M_R \sim 10^{14} \text{ GeV} \sim M_{GUT}$$

$$\begin{pmatrix} 0 & m_D^T \\ m_D & M_R \end{pmatrix}$$

- Can small neutrino masses due to low (TeV) scale physics?
- new EW models (littel Higgs, Higgsless models, etc):  $\Lambda \sim \text{TeV}$
- can be tested at colliders

# An Alternative

- gauge symmetry SM x non-anomalous  $U(1)_\nu + N_\nu$
- SM particles &  $\nu_R$  : all charged under  $U(1)$
- $U(1)$  forbids dim-4, dim-5 operators
- To get  $m_\nu \neq 0$  :  ~~$U(1)_\nu$~~   $\langle \phi \rangle$  : SM singlet breaks  $U(1)_\nu$

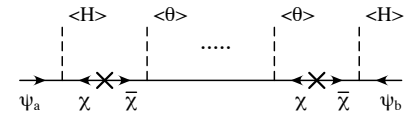
$$\frac{HH\ell\ell}{M} \left( \frac{\langle \phi \rangle}{\Lambda} \right)^p$$

,  $p$  large

$$\langle \phi \rangle < \Lambda,$$

$$\frac{\langle \phi \rangle}{\Lambda} \sim 10^{-1}$$

$$\Lambda \sim \text{TeV!}$$



- non-anomalous  $U(1)_\nu$ 
  - anomaly cancellation conditions  $\Rightarrow$  constraints on  $U(1)$  charges
  - generation dependent charges  $\Rightarrow U(1)$  flavor symmetry
    - $\Rightarrow$  mixing pattern & mass hierarchy (FN)
  - light sterile neutrinos: DM candidate
  - TeV scale  $Z'$ : probing flavor sector at colliders
- c.f. anomalous  $U(1)$ : mixed anomaly cancelled by Green-Schwarz mechanism
  - $U(1)$  broken at fundamental string scale
  - $[U(1)]^3$  anomaly cancelled by other exotics

constraints not as stringent

# The Model

SM x  $U(1)_\nu$  + N  $\nu_R$

- generation independent quark charges  $\Rightarrow$  no FCNC in quark sector

$$q_L^i : z_q \quad u_R^i : z_u \quad d_R^i : z_d$$

- generation dependent lepton charges  $\Rightarrow$  neutrino mixing

$$\ell_L^i : z_{\ell_i} \quad e_R^i : z_{e_i} \quad \nu_R^k : z_{n_k}$$

- scalar charges:

$$H : z_H \quad \phi : \text{SM singlet } +1, \quad \langle \phi \rangle \text{ breaks } U(1)_\nu$$

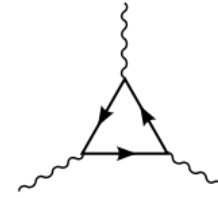
- N = 1, 2: always terms allowed at dime-4  
 $\Rightarrow$  high seesaw scale
- lowest value for N to get TeV scale seesaw: N=3
- with N=3: (9+N) charges

# Anomaly Cancellation

$SU(3) \times SU(2)_L \times U(1)_Y \times U(1)_\nu$

- 4 mixed anomalies not involving  $\nu_R$

$$\begin{aligned} [SU(3)]^2 U(1)_\nu &: & z_d &= 2z_q - z_u \\ U(1)_\nu [SU(2)_L]^2 &: & \sum_{i=1}^3 z_{\ell_i} &= -9z_q \\ U(1)_\nu [U(1)_Y]^2 &: & \sum_{i=1}^3 z_{e_i} &= -3(2z_q + z_u) \\ U(1)_Y [U(1)_\nu]^2 &: & \sum_{i=1}^3 (z_{\ell_i}^2 - z_{e_i}^2) &= 3(z_q - z_u)(5z_q + z_u) \end{aligned}$$



- 2 anomaly conditions involving  $\nu_R$

$$\begin{aligned} U(1)_\nu - \text{graviton} &: & \sum_{k=1}^N z_{n_k} &= -3(4z_q - z_u) \\ [U(1)_\nu]^3 &: & \sum_{i=1}^3 z_{n_k}^3 &= (2z_{\ell_i}^3 - z_{e_i}^3) - 54z_q(z_q - z_u)^2 \end{aligned}$$

- additional constraints:

- top mass generated at dim-4:  $z_H = z_u - z_q$   
(all quark masses allowed at dim-4)
- diagonal elements in  $M_e$  allowed at dim-4

$$\lambda_e^{ii} \bar{\ell}_L^i e_R^i H$$

$$m_e \sim \begin{pmatrix} \lambda_{11} & * & * \\ * & \lambda_{22} & * \\ * & * & \lambda_{33} \end{pmatrix}$$

- charged fermion sector: 4 independent charges
- neutrino sector: (N-2) independent charges

# Neutrino Mass Terms

U(1)<sub>v</sub> forbids  $H\bar{\ell}_L\nu_R$ ,  $\frac{HH\bar{\ell}_L^c\ell_L}{\Lambda}$ ,  $M_R\bar{\nu}_R^c\nu_R$ ; after its breaking:

- LH Majorana mass terms:

$$\sum_{i,j} \frac{c_{\ell}^{ij}}{\Lambda} \left(\frac{g_{\phi}}{\Lambda}\phi\right)^{q_{ij}} \bar{\ell}_L^i \ell_L^j HH \quad q_{ij} = 2(z_q - z_u) - z_{\ell_i} - z_{\ell_j}$$

- Dirac mass terms:

$$\sum_{i,k} \lambda_{\nu}^{ik} \left(\frac{g_{\phi}}{\Lambda}\phi\right)^{p_{ik}} \bar{\ell}_L^i n_R^k \tilde{H} \quad p_{ik} = z_u - z_q + z_{\ell_i} - z_{n_k}$$

- RH Majorana mass terms:

$$\sum_{k,k'} c_n^{kk'} \Lambda \left(\frac{g_{\phi}}{\Lambda}\phi\right)^{r_{kk'}} \bar{n}_R^k n_R^{k'} \quad r_{kk'} = -z_{n_k} - z_{n_{k'}}$$

- resulting neutrino mass matrix:

$$M_{\nu} \sim \left( \begin{array}{c|c} \frac{v^2}{\Lambda} \epsilon^{|q|} & v \epsilon^{|p|} \\ \hline v(\epsilon^{|p|})^{\top} & \Lambda \epsilon^{|r|} \end{array} \right)$$

- pure type-I seesaw unattainable

$$q_{ij} = -p_{ik} - p_{jk'} + r_{kk'} = \text{integer}$$

-- Pure Dirac  
-- Type-II seesaw

# Charged Lepton Sector

- charged lepton masses:

$$\sum_{ij} \lambda_e^{ij} \left( \frac{g_\phi}{\Lambda} \phi \right)^{s_{ij}} \bar{\ell}_L^i e_R^j H \quad s_{ij} = z_{\ell_i} - z_{\ell_j}$$

- generation-dependent U(1) charges:

$$s_{ij} \neq 0 \quad \text{for} \quad i \neq j \quad \Rightarrow \quad \text{tree-level FCNC mediated by } Z'$$

$$\bar{f} V^\dagger \gamma_\mu \begin{pmatrix} z_{\ell_1} & & \\ & z_{\ell_2} & \\ & & z_{\ell_3} \end{pmatrix} V f$$

- flavor changing processes

$$\mu \rightarrow e, \quad \mu^+ \rightarrow e^+ e^- e^+, \quad \tau \rightarrow \ell \ell' \ell''$$

$$\text{decay rate: } \Gamma \propto \left( \frac{\epsilon^{|s_{ij}|}}{\langle \phi \rangle^2} \right)^2, \quad \epsilon^{|s_{ij}|} < 1$$

# Leptocratic Models

- with N=3: 5 independent charges  $z_q, z_{\ell_1}, z_{\ell_2}, 2z_{n_k}$
- Cubic equation: rational solutions non-trivial [Fermat's Last Theorem]

$$\begin{aligned}
 z_{\ell_1} &\equiv -3z_q - 2a, & z_{e_1} &\equiv -6z_q + \frac{c}{2} - 2a, & z_{n_1} &\equiv -\frac{c}{2} - 2b, \\
 z_{\ell_2} &\equiv -3z_q + a + a', & z_{e_2} &\equiv -6z_q + \frac{c}{2} + a + a', & z_{n_2} &\equiv -\frac{c}{2} + b + b', \\
 z_{\ell_3} &\equiv -3z_q + a - a', & z_{e_3} &\equiv -6z_q + \frac{c}{2} + a - a', & z_{n_3} &\equiv -\frac{c}{2} + b - b',
 \end{aligned}$$

- $[U(1)_V]^3$  : general class of rational solutions

$$c = -2 \frac{a(a^2 - a'^2) - b(b^2 - b'^2)}{3a^2 + a'^2 - 3b^2 - b'^2}$$



# Leptocratic Models

- resulting mass matrices

- Dirac neutrino mass matrix

$$p = \begin{pmatrix} -2(a-b) & -(2a+b+b') & -(2a+b-b') \\ a+a'+2b & a+a'-b-b' & a+a'-b+b' \\ a-a'+2b & a-a'-b-b' & a-a'-b+b' \end{pmatrix}$$

- LH Majorana mass matrix

$$q = \begin{pmatrix} c+4a & c+a-a' & c+a+a' \\ c+a-a' & c-2(a+a') & c-2a \\ c+a+a' & c-2a & c-2(a-a') \end{pmatrix}$$

- RH Majorana mass matrix

$$r = \begin{pmatrix} c+4b & c+b-b' & c+b+b' \\ c+b-b' & c-2(b+b') & c-2b \\ c+b+b' & c-2b & c-2(b-b') \end{pmatrix}$$

- Charged lepton mass matrix

$$s = \begin{pmatrix} 0 & -3a-a' & -3a+a' \\ 3a+a' & 0 & 2a' \\ 3a-a' & -2a' & 0 \end{pmatrix}$$

# Orwellian Leptocratic Model

- ▶ all  $U(1)_{NA}$  charges for SM fermions are generation independent:
  - ▶  $a = a' = b' = 0; \quad c = -2b/3$
  - ▶ no charged lepton flavor violating FCNC mediated by  $Z'$  at tree level
  - ▶  $Q(N_2) = Q(N_3) \neq Q(N_1)$
  - ▶ bi-large mixing through anarchy
- ▶ three active neutrinos: can either be Dirac or Majorana fermions

**(I) Dirac neutrinos:**  $b = \text{integer}$ , but not  $(3n) \Rightarrow c = \text{non-int}$ , only  $M_D$  allowed

$$M_D = v \epsilon^{|b|} \begin{pmatrix} \lambda_\nu^{11} \epsilon^{|b|} & \lambda_\nu^{12} & \lambda_\nu^{13} \\ \lambda_\nu^{21} \epsilon^{|b|} & \lambda_\nu^{22} & \lambda_\nu^{23} \\ \lambda_\nu^{31} \epsilon^{|b|} & \lambda_\nu^{32} & \lambda_\nu^{33} \end{pmatrix} \quad \Lambda \sim 1 \text{ TeV}$$

$$\lambda \sim 10^{-4} - 10^{-5} : \quad b = \pm 2$$

$$\lambda \sim 1 : \quad b = \pm 13$$

# Orwellian Leptocratic Model

- ▶ (II) Majorana neutrinos:  $b = (3n) \Rightarrow M_D, M_{LL}, M_{RR}$  all allowed

$$M_L = \frac{v^2}{\Lambda} \epsilon^{2|b|/3} c_\ell \quad M_R = \Lambda \epsilon^{|b|/3} \begin{pmatrix} c_n^{11} \epsilon^{3|b|} & c_n^{12} & c_n^{13} \\ c_n^{12} & c_n^{22} \epsilon^{7|b|/3} & c_n^{23} \epsilon^{7|b|/3} \\ c_n^{13} & c_n^{23} \epsilon^{7|b|/3} & c_n^{33} \epsilon^{7|b|/3} \end{pmatrix}$$

- if  $\lambda \sim c$ :  $M_{LL} > M_D \Rightarrow$  inverted seesaw

- for  $\lambda \sim 10^{-5}$ ,  $\epsilon \sim 10^{-4}$ ,  $|b| = 3$

$$\Rightarrow \text{light sterile } m_{\nu_4} \sim \Lambda \epsilon^{4|b|/3} \left( \frac{\lambda_\nu^2}{c_\ell} \right) \sim 10^{-9} \text{ eV}$$

$$\Theta_{\text{active-light}} \sim \epsilon \frac{\Lambda}{v} \sim 10^{-3}$$

$$\Rightarrow 2 \text{ heavy sterile } m_{\nu_{5,6}} \sim \Lambda \epsilon^{|b|/3} c_n \sim 1 \text{ keV}$$

$$\Theta_{\text{active-heavy}} \sim \epsilon^2 \frac{v}{\Lambda} \sim 10^{-9}$$

- for  $\lambda \sim 1$ ,  $\epsilon \sim 0.1$ ,  $|b| = 18$

$$\text{light sterile: } m_{\nu_4} \sim 10^{-12} \text{ eV}$$

$$\Theta_{\text{active-light}} \sim \epsilon^6 \frac{\Lambda}{v} \sim 10^{-5}$$

$$\text{heavy sterile: } m_{\nu_{5,6}} \sim 1 \text{ MeV}$$

$$\Theta_{\text{active-heavy}} \sim \epsilon^{12} \frac{v}{\Lambda} \sim 10^{-13}$$

# (2+1) Leptocratic Model

- ▶ generation dependent charged lepton charges

$$z_{l_1} \neq z_{l_2} = z_{l_3}$$

$$z_{e_1} \neq z_{e_2} = z_{e_3}$$

$$z_{n_1} \neq z_{n_2} = z_{n_3}$$

- ▶  $a' = b' = 0$ ;  $a, b \neq 0$

$$c = -\frac{2}{3} \left( \frac{a^2 + ab + b^2}{a + b} \right)$$

- ▶ large neutrino mixing from U(1) symmetry
- ▶  $a = 25/3$ ,  $b = -11/3$ ,  $\epsilon \sim 0.1 \Rightarrow$  pure Dirac

$$M_D = v \epsilon^{12} \begin{pmatrix} \mathcal{O}(\epsilon^{12}) & \mathcal{O}(\epsilon) & \mathcal{O}(\epsilon) \\ \mathcal{O}(\epsilon^{13}) & \mathcal{O}(1) & \mathcal{O}(1) \\ \mathcal{O}(\epsilon^{13}) & \mathcal{O}(1) & \mathcal{O}(1) \end{pmatrix}$$

- ▶ LFV in charged lepton sector?

$|s_{12}| = |s_{23}| = 3a = 25$ : suppressed by  $10^{-25}$  Safe!

$z_{l_2} = z_{l_3}$  : no tree-level FCNC

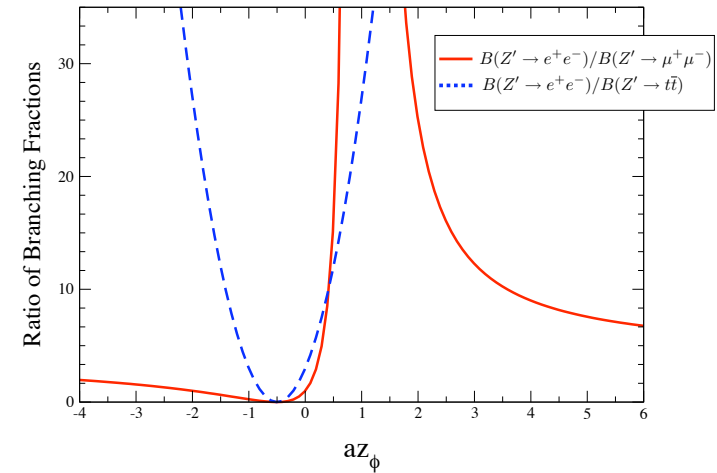
# TeV Scale Seesaw

- probing the flavor sector at the colliders
- (2+1) leptocratic models:

$$\frac{B(Z' \rightarrow e^+e^-)}{B(Z' \rightarrow \mu^+\mu^-)} = \left( \frac{1 + 2az_\phi}{1 - az_\phi} \right)^2$$

$$\frac{B(Z' \rightarrow e^+e^-)}{B(Z' \rightarrow t\bar{t})} = 3(1 + 2az_\phi)^2$$

$$z_\phi = -\frac{3(a+b)}{a^2 + ab + b^2}$$



- invisible decays of  $Z'$ : distinguish different  $U(1)$ 
  - $U(1)_{B-L}$ :  $B(Z' \rightarrow \text{invisible}) = 3/8$
  - Orwellian  $Z'$ :  $B(Z' \rightarrow \text{invisible}) = 6/7$

$$[z_{n1} = 5, z_{n2} = z_{n3} = -4]$$

# Conclusion

- TeV scale seesaw:
  - required in new EW models; testability at colliders
- SM x non-anomalous U(1), in presence of RH neutrinos
- anomaly cancellations: constraints on charges, predict flavor structure
- TeV scale seesaw possible with 3 RH neutrinos
- neutrinos can either be Dirac or Majorana fermions
- Orwellian leptocratic models: generation independent charged lepton charges
- (2+1) leptocratic models: generation dependent charged lepton charges
- GUT extension:  $SU(5) \times U(1)_{NA}$  [MCC, Jones, Rajaraman, Yu, arXiv:0801.4228]
  - family symmetry
  - proton stability