Comparing the Higgs Sector of Two HDM with the Scalar Sector of Linear Sigma Model with two Nonets

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What Is Our Aim? Plan of the Talk

- For heavy Higgs, how we calculate scattering amplitude of $W^+W^- \to W^+W^-$.
- Plan of the Tlak
  - Correspondence Between $\sigma\boldsymbol{\pi}$ $SU(2)_L \times SU(2)_R$Linear Sigma Model and the Higgs Sector of $SU(2)_L \times U(1)_Y$ EW theory.
  - K Matrix and LSM Unirization
  - Application to the SU(2)xU(1) Electroweak Model and Equivelence Theorem
    $\mathrm{amp}(W_L^+W_L^- \to W_L^+W_L^-) = \mathrm{amp}(\pi^+\pi^- \to \pi^+\pi^-) + \mathcal{O}(\frac{m_W}{E_W})$
  - Correspondence between $\sigma\boldsymbol{\pi}$, $\sigma^{'}\boldsymbol{\pi}^{'}SU(2)_L \times SU(2)_R$Linear Sigma Model and the 2HDM.
  - $\mathrm{amp}(W_L^+W_L^- \to W_L^+W_L^-) = \mathrm{amp}(\pi^+\pi^- \to \pi^+\pi^-) + \mathcal{O}(\frac{m_W}{E_W})$ in 2HDM with Custodial Symmetry (Under Progress)
  - Summary

Linear Sigma Model and the Higgs Sector of SM

- In low Energy QCD the lagrangian is

$$\mathcal{L} = \frac{1}{2}\left(\partial_\mu \boldsymbol{\pi} \cdot \partial^\mu \boldsymbol{\pi} + \partial_\mu \sigma \partial^\mu \sigma\right) - V(\sigma, \boldsymbol{\pi}\ ), \tag{1}$$

$$V(\sigma, \boldsymbol{\pi}\ ) = -\frac{\mu^2}{2}\left(\sigma^2 + \boldsymbol{\pi}\ ^2\right) + \frac{\lambda}{4}\left(\sigma^2 + \boldsymbol{\pi}\ ^2\right)^2 \tag{2}$$

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  - where sing of λ are choosen to ensure SSB, and
    $\mathrm{F}_\pi = \sqrt{2}\,\langle\sigma\rangle\,;\ \ \mu^2 = \frac{1}{2} m_{\sigma b}^2,\ \ \lambda = \frac{m_{\sigma b}^2}{2\langle\sigma\rangle^2}$

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  - Lagrangian, Eq. (1) is written

$$\mathcal{L} = \partial_\mu \Phi^\dagger \partial^\mu \Phi + \mu^2 \Phi^\dagger \Phi - \lambda\,\left(\Phi^\dagger \Phi\right)^2. \tag{4}$$

Here as notation $v = \langle\sigma\rangle = \frac{F_\pi}{\sqrt{2}} = 0.0655$. In the EW theory, $v = 0.246$ TeV, about 2656 times the value in the low energy QCD

Unitarized LSM

- The contribution for $\pi^+\pi^- \rightarrow \pi^+\pi^-$coming from contact term, s channel and the crossed Higgs boson exchange.
- The I=J=0 partial wave amplitude at tree level is

$$\left[T_0^0\right]_{\text{tree}}(s) = \alpha(s) + \frac{\beta(s)}{m_{\sigma b}^2 - s} \tag{5}$$

where

$$\begin{aligned} \alpha(\mathrm{s}) &= \frac{\sqrt{1-\frac{4m_\pi^2}{s}}}{32\pi F_\pi^2}\left(m_{\sigma b}^2 - {m_\pi}^2\right) \times \\ &\quad \left[-10 + 4\frac{m_{\sigma b}^2 - {m_\pi}^2}{s - 4{m_\pi}^2}\ln\left(\frac{\mathrm{m}_{\sigma \mathrm{b}}^2 + \mathrm{s} - 4{\mathrm{m}_\pi}^2}{\mathrm{m}_{\sigma \mathrm{b}}^2}\right)\right], \end{aligned} \tag{6}$$

$$\beta(s) = \frac{3\sqrt{1-\frac{4m_\pi^2}{s}}}{16\pi F_\pi^2}\left(m_{\sigma b}^2 - {m_\pi}^2\right)^2. \tag{7}$$

Unitarized LSM

- The S matrix given by

$$S_0^0(s) = 1 + 2iT_0^0(s). \tag{8}$$

has some problems the Amplitude diversge at $s = m_{\sigma b}^2$.

- One solution is

$$\frac{1}{m_{\sigma b}^2 - s} \longrightarrow \frac{1}{m_{\sigma b}^2 - s - im_{\sigma b}\Gamma}. \tag{9}$$

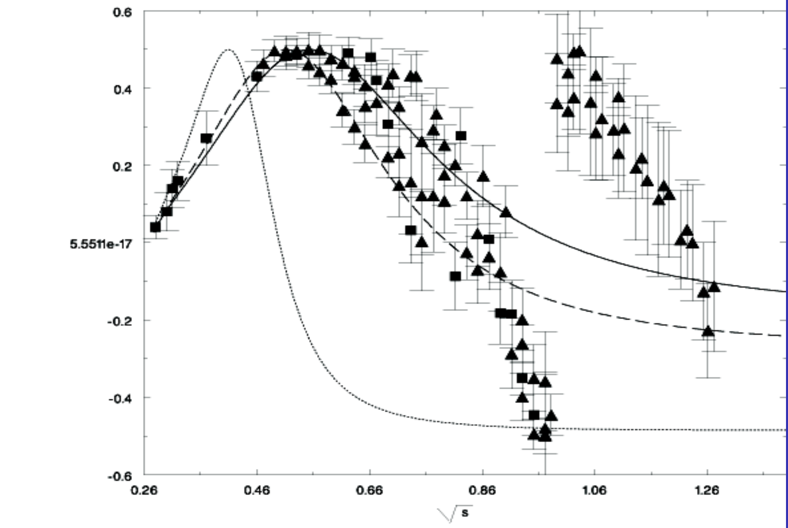
- We use K-Matrix unitrization

$$S_0^0(s) = \frac{1 + i\left[T_0^0\right]_{\text{tree}}(s)}{1 - i\left[T_0^0\right]_{\text{tree}}(s)} \tag{10}$$

- Hence

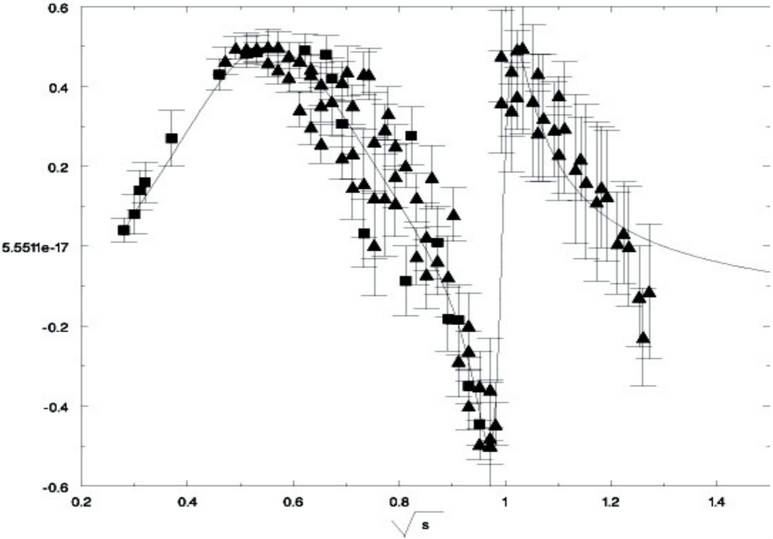
$$T_0^0(s) = \frac{\left[T_0^0\right]_{tree}(s)}{1 - i\left[T_0^0\right]_{tree}(s)}. \tag{11}$$

Unitarized LSM



Unitarized LSM

- Including the third flavour



Physicsl Mass and Width of Sigma

- How nonperturpative LSM in low energy QCD

$$\lambda = \frac{m_{\sigma b}^2}{2v^2} \gg 1. \qquad (12)$$

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- In a non-perturbative regime one might expect the physical parameters like the sigma mass and width to differ from their "bare" or tree-level values.
  To see this we look at the σ pole in

$$T_0^0(s) = \frac{(m_{\sigma b}^2 - s)\alpha(s) + \beta(s)}{(m_{\sigma b}^2 - s)[1 - i\alpha(s)] - i\beta(s)}. \tag{13}$$

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- The pole position z_0 is then given as the solution of:

$$(m_{\sigma b}^2 - z_0)[1 - i\alpha(z_0)] - i\beta(z_0) = 0. \tag{14}$$

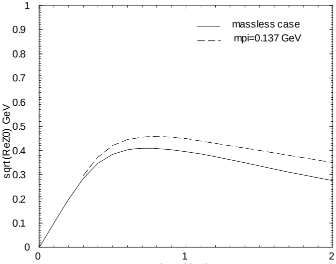
Physicsl Mass and Width of Sigma

For treating both the low energy QCD as well as the standard electroweak situation it is convenient to introduce the scaled quantities :

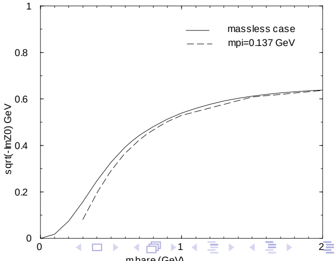
$$\bar{m} = \frac{m_{\sigma b}}{F_\pi} = \frac{m_{\sigma b}}{\sqrt{2}v}; \quad \bar{z}_0 = \frac{z_0}{F_\pi^2} = \frac{z_0}{2v^2}. \tag{15}$$

We have $\bar{z}_0 \approx \frac{352}{3}\frac{\pi^2}{\bar{m}^2} - 8\pi i$. Fitting; $m_{\sigma b} \simeq 0.85$ GeV, $\bar{m} \simeq 6.5$; $m_{\sigma-physical} \simeq 0.46$ GeV

Figs. 3



Figs. 4



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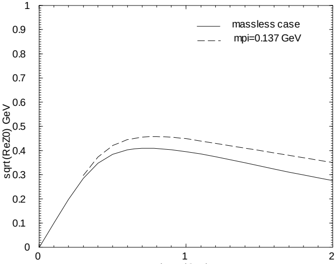
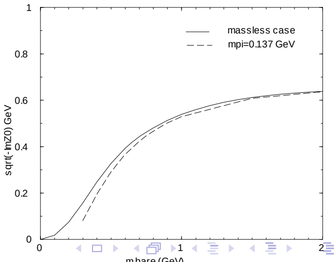


Figs. 4

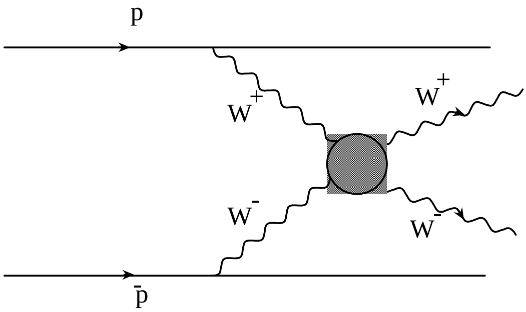


Application to the SU(2)xU(1) Electroweak Model

- Lagrangian Eq. (1), characterized by the scale $v = \frac{0.131}{\sqrt{2}}$ GeV. Same Lagrangian Eq. (4), characterized by the scale $v = 0.246$ TeV
- Clearly it is desirable to consider a model, like the present one, which has the possibility of describing the scattering amplitude around the energy of the Higgs boson even if it were to exist in a non-perturbative scenario.
- The discussion of the $\pi\pi$ scattering amplitude T_0^0 , above, can also be used to treat the high energy scattering of the longitudinal components of the W and Z bosons in the electroweak theory by making use of the Goldstone boson equivalence theorem
- Goldstone boson equivalence theorem states that: at high energy the amplitude of longtudinally massive gauge bosons equal to the amplitude of the Goldstone boson that was eatean by the gause boson. Here $(W_L^+ W_L^-$ and $\pi^+\pi^-)$

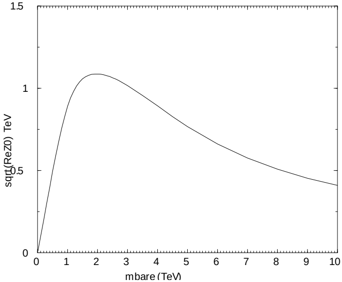
Application to the SU(2)xU(1)

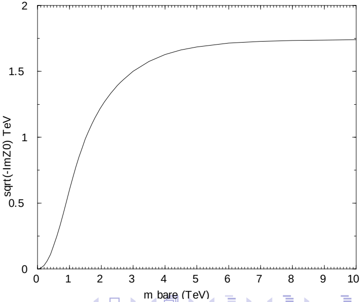
- $$\begin{aligned} \mathrm{amp}(W_L^+ W_L^- \to W_L^+ W_L^-) &= \mathrm{amp}(\pi^+\pi^- \to \pi^+\pi^-) + \mathcal{O}(\frac{m_W}{E_W}) \\ \mathrm{amp}(W_L^+ W_L^- \to Z_L Z_L) &= \mathrm{amp}(\pi^+\pi^- \to \pi^0\pi^0) + \mathcal{O}(\frac{m_W}{E_W}) \end{aligned}$$



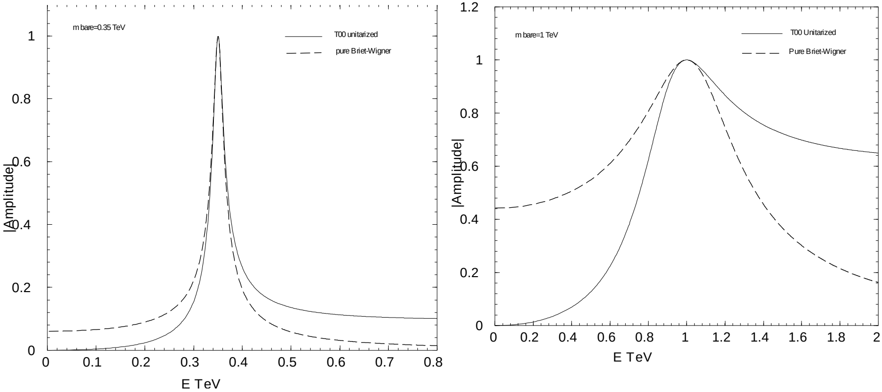
Application to the SU(2)xU(1)

- The Higgs pole positions can be gotten from Figs. 3 and 4 using the scaled quantities defined in Eq. (15). $m_{\sigma b} \simeq 0.85$ GeV, $\bar{m} \simeq 6.5$. This value of $\bar{m}$ corresponds to a bare Higgs mass value of $m_{\sigma b} = 2.26$ TeV. At that value, the measure of the physical Higgs mass, $\sqrt{\mathrm{Re}(z_0)}$ would be about 1.1 TeV and $\sqrt{-\mathrm{Im}(z_0)}$ would be about 1.3 TeV





Amplitudes in K matrix and Briet-Wigner schemes



2HDM

The most general form for the potential of 2HDM is

$$
\begin{aligned}
V(\Phi_1, \Phi_2) = {} & m_{11} \left|\Phi_1\right|^2 + m_{22} \left|\Phi_2\right|^2 + (\beta_1 \Phi_1^* \Phi_2 + h.c.) + \\
& + \lambda_1 \left|\Phi_1\right|^4 + \lambda_2 \left|\Phi_2\right|^4 + \lambda_3 \left|\Phi_1\right|^2 \left|\Phi_2\right|^2 \\
& + [\beta_2 (\Phi_1^* \Phi_2)^2 + h.c.] + [\beta_3 (\Phi_1^* \Phi_2) \left|\Phi_1\right|^2 + h.c.] \\
& + [\beta_4 (\Phi_1^* \Phi_2) \left|\Phi_2\right|^2 + h.c.] + \lambda_4 (\Phi_1^* \Phi_2)(\Phi_2^* \Phi_1).
\end{aligned}
$$

It has 14 parameters (6 real + 4 complex). with 2 types of dimensions.

- Some symmetries can be imposed to reduce the number of parameters.
- For example Z_2 symmetry $\Phi_1 \rightarrow \Phi_1$, $\Phi_2 \rightarrow -\Phi_2$, hence terms contains odd power of Φ_2 wil not appear, we end up with 6 parameters.
- Z_2 symmetry avoids FCNC
- Hard violation of Z_2 means $\beta_3 \neq 0, \beta_2 \neq 0$
- Soft violation of Z_2 $(\beta_2 = \beta_3 = 0)$.

LSM via 2HDM

- Let $\Phi_i = \begin{pmatrix} i\pi_i^+ \\ \frac{\sigma_i - i\pi_i^0}{\sqrt{2}} \end{pmatrix}$ *and* $\mathbf{M}_i = \sigma_i I + i\tau \cdot \boldsymbol{\pi}_i$

  $\mathbf{M}_i$ can take the form $\mathbf{M}_i = (i\tau_2\Phi_i^* \;\; \Phi_i)$
- We have
- $tr(\mathbf{M}_1\mathbf{M}_1^\dagger) = \mathbf{4}\;\Phi_1^\dagger\Phi_1$
- $tr(\mathbf{M}_2\mathbf{M}_2^\dagger) = \mathbf{4}\;\Phi_2^\dagger\Phi_2$
- $tr(\mathbf{M}_1\mathbf{M}_2^\dagger) = tr(\mathbf{M}_2\mathbf{M}_1^\dagger) = \mathbf{4}\;\mathrm{Re}(\Phi_1^\dagger\Phi_2)$
- $\mathbf{M}_i$ transforms as $\mathbf{M}_i \to U_L \mathbf{M}_i U_R^\dagger$
- In taht case the The QCD Lagrangian has custodial symmetry, meaning no $\mathrm{Im}(\Phi_1^\dagger\Phi_2)$ term.

LSM and 2 HDM

- The QCD Lagrangian that describes σ_i and $\boldsymbol{\pi}_i$ **is**

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}\left(\partial_\mu M_1 \partial^\mu M_1^\dagger\right) + \frac{1}{2}\mathrm{Tr}(\partial_\mu M_2 \partial^\mu M_2^\dagger) - V\left(M_1, M_2\right), \tag{17}$$

where

$$\begin{aligned} V\left(M_1, M_2\right) &= \alpha_1 I_1 + \alpha_2 I_2 + \alpha_3 I_3 + \alpha_4 I_1^2 + \alpha_5 I_2^2 + \alpha_6 I_3^2 \\ &\quad + \alpha_7 I_1 I_2 + \alpha_8 I_1 I_3 + \alpha_9 I_2 I_3 \end{aligned} \tag{18}$$

$$I_1 = \frac{1}{2}\mathrm{Tr}\left(M_1 M_1^\dagger\right) = \sigma_1^2 + \boldsymbol{\pi}_1^2 \tag{19}$$

$$I_2 = \frac{1}{2}\mathrm{Tr}\left(M_2 M_2^\dagger\right) = \sigma_2^2 + \boldsymbol{\pi}_2^2 \tag{20}$$

$$I_3 = \frac{1}{2}\mathrm{Tr}\left(M_1 M_2^\dagger\right) = \sigma_1 \sigma_2 + \boldsymbol{\pi}_1 \boldsymbol{\pi}_2 \tag{21}$$

LSM and 2 HDM

- We can impose Z_2 symmetry $M_1 \to M_1$, $M_2 \to -M_2$.
- This equivelent to Parity Conservation in QCD.
- For example take $\sigma_2 = \eta$ and $\boldsymbol{\pi}_2 = \mathbf{a}$ hence $\Phi_2 = \begin{bmatrix} -ia^+ \\ \frac{\eta + ia^0}{\sqrt{2}} \end{bmatrix}$
- In that case the low energy QCD Lagrangian should conserve partity, hence

$$\begin{aligned} V(M_1, M_2) &= \alpha_1 I_1 + \alpha_2 I_2 + \alpha_4 I_1^2 + \alpha_5 I_2^2 + \alpha_6 I_3^2 \\ &\quad + \alpha_7 I_1 I_2 \end{aligned} \tag{22}$$

- After straightforward calculations

LSM and 2 HDM

- Easy to see

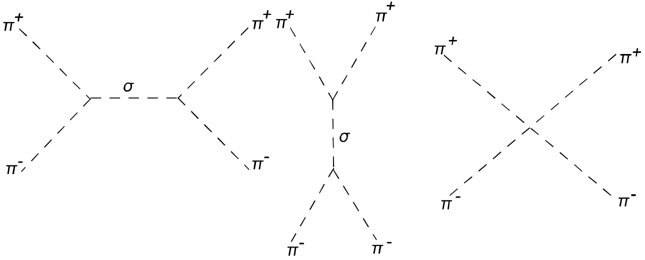
$$\langle \pi_i \rangle = 0; \quad \langle a_i \rangle = 0; \quad \langle \sigma^2 \rangle = -\frac{\alpha_1}{\alpha_3} \tag{23}$$

- Masses

$$\langle \frac{\partial^2 V}{\partial \sigma^2} \rangle = 8\alpha_3 \langle \sigma \rangle^2; \quad \langle \frac{\partial^2 V}{\partial \eta^2} \rangle = 2\alpha_2 + 2(\alpha_5 + \alpha_6) \langle \sigma \rangle^2;$$
$$\langle \frac{\partial^2 V}{\partial \pi_i \partial \pi_j} \rangle = 0; \; \langle \frac{\partial^2 V}{\partial a_i \partial a_j} \rangle = 2\delta_{ij} [\alpha_2 + \alpha_6 \langle \sigma \rangle^2] \tag{24}$$

LSM and 2 HDM

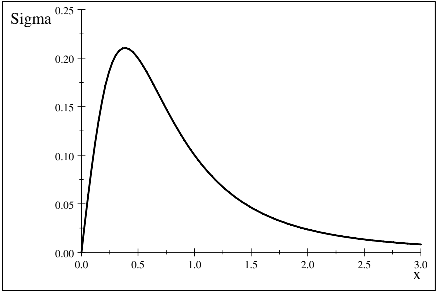
- In case of $\langle \eta \rangle = 0$ we have the following contributions



The unitized amplitude

- $A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{x[x - i(1+2x^2)]}{(1+x^2)(1+4x^2)}$ where $x = \frac{s}{16\pi v^2}$
- The Cross section for this process is

$$\sigma(W_L^+ W_L^- \rightarrow W_L^+ W_L^-) = \frac{1}{v^2} \frac{x[x^2 + (1+2x^2)^2]}{(1+x^2)^2(1+4x^2)^2} \qquad (25)$$


Sigma
0.25
0.20
0.15
0.10
0.05
0.00
0.0
0.5
1.0
1.5
2.0
2.5
3.0
x

LSM and 2 HDM Uder Progress

- In case of $\langle \eta \rangle \neq 0$ more terms will be included
  Uder progress

Summary

- If LHC didn't detect SM Higgs, then Higgs could be very heavy.
- For heavy Hggs WW interactions become strong at TeV scales.
- It could be possiblity for resonance or some new physics
- We gave a comaprsion between scalar sectot of LSM and EW Higgs sector in Case of 1 and 2 HD.
- LSM of 2 nonets has Custodial symmetry by its construction
- K matix usnirization has been used to get $A(W_L^+ W_L^- \rightarrow W_L^+ W_L^-)$
- More results are under progress.