Generation Symmetry and E_6 Unification

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Outline

- Some Motivations: Observed Hierarchies
  Neutrino Data

- E_6 GUT and Generation symmetry – $SO(3)_g$

- My focus and aim:
  Relate fermion masses & mixings
  Including (new) heavy GUT states;
  Explore the role of the *symmetries*

- Charged fermion masses & mixings

Observed Noticeable Hierarchies:

$$\lambda_t \sim 1 \ , \qquad \lambda_u : \lambda_c : \lambda_t \sim \lambda^8 : \lambda^4 : 1$$

$$\lambda_b \sim \lambda_\tau \sim \frac{m_b}{m_t} \tan\beta \ , \qquad \lambda_d : \lambda_s : \lambda_b \sim \lambda^4 : \lambda^2$$

With λ=0.2

$$\lambda_e : \lambda_\mu : \lambda_\tau \sim \lambda^5 : \lambda^2 : 1$$

$$V_{us} \approx \lambda \ , \qquad V_{cb} \approx \lambda^2 \ , \qquad V_{ub} = \lambda^4 - \lambda^3$$

What is origin of these hierarchies?
Is there any relation or sum rule?
Why three families?

Within SM no answer to these questions…

Atmospheric & Solar Neutrino Data

$$\Delta m^2_{\rm atm} = 2.4 \cdot 10^{-3} {\rm eV}^2$$

$$\Delta m^2_{\rm sol} = 7.9 \cdot 10^{-5} {\rm eV}^2$$

$$\sin^2 \theta_{23} = 0.44$$

$$\sin^2 \theta_{12} = 0.314$$

Third mixing angle: $\theta_{13} \leq 0.2$ **(CHOOZ bound)**

Unknown phase: $\delta_{\rm lept}$

are of great importance for leptonic CP viol.

- **Origin of these scales and mixings?**

Unexplained in SM/MSSM

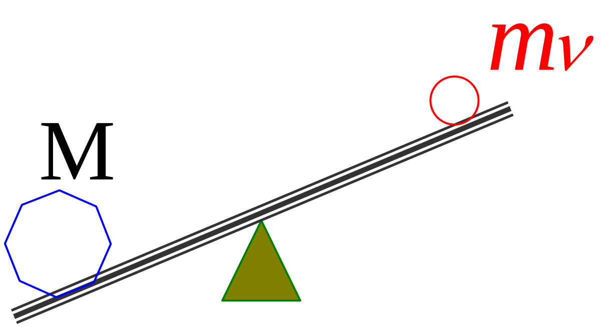
Grand Unification
Answers to some of those questions & offers solutions to some problems/puzzles

- **Matter Unification**

In $SO(10)$: $(q,\ u^c,\ e^c,\ d^c,\ l, \nu_R) = \mathbf{16}$

In E_6: $(\nu^c + \text{all matter} + \textit{new}\ \text{states}) = 27$

SO(10), E6 → ν_R Neutrino masses via see-saw → Oscillations



And interesting asymptotic relations:

$$\lambda_t = \lambda_b = \lambda_\tau \quad ,\ldots$$

$$m_{\nu D} = m_t$$

Interesting Properties of E6

SU(2)L x U(1)Y x SU(3)C --> SU(3)L x SU(3)R x SU(3)C $\equiv SU(3)^3$

$Q_L(x) = (3, 1, \bar{3})$, $L(x) = (\bar{3}, 3, 1)$, $Q_R(x) = (1, \bar{3}, 3)$

$\cup$

ν_R, N Right handed neutrinos

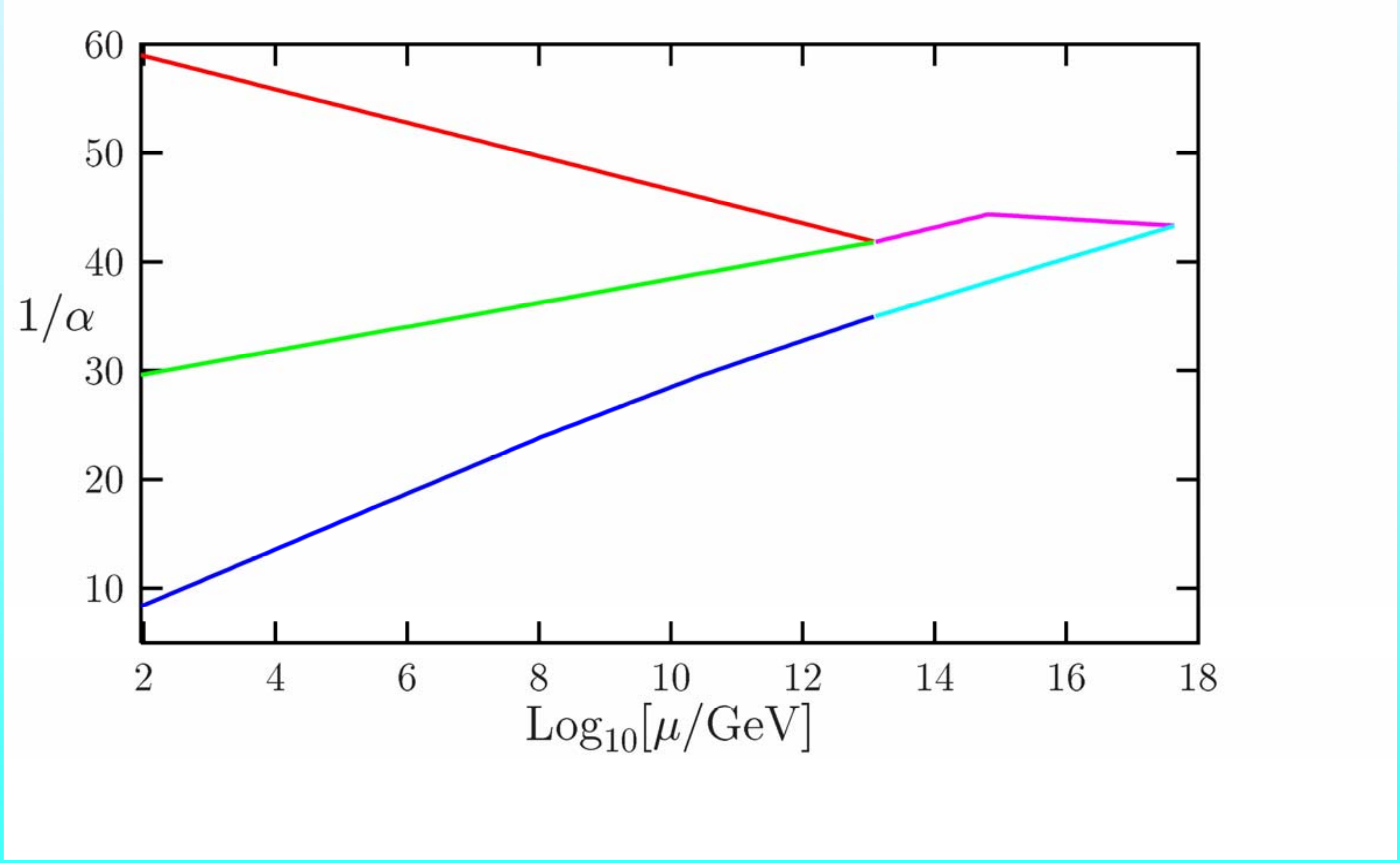
$SU(3)^3 \subset E6$ -- minimal exceptional group

QL+QR+L → ψ(27)

And intermediate $SU(3)^3$ for coupling unification

Unification with E6

`Concorde'



$E_6 \to SU(3)_L \times SU(3)_R \times SU(3)_C$

$27 = L(3^*,3,1)+Q_L(3,1,3^*)+Q_R(1,3^*,3)$

single generation for fermions $\psi(27)$

$$(Q_L)^a_i = \begin{pmatrix} u^a \\ d^a \\ D^a \end{pmatrix}, \quad L^i_k = \begin{pmatrix} L^1_1 & E^- & e^- \\ E^+ & L^2_2 & \nu \\ e^+ & \hat{\nu} & L^3_3 \end{pmatrix}, \quad (Q_R)^k_a = \left(\hat{u}_a,\ \hat{d}_a,\ \hat{D}_a\right)$$

$$Q_L(x) = (3,1,\bar{3})\ , \qquad L(x) = (\bar{3},3,1)\ , \qquad Q_R(x) = (1,\bar{3},3)$$

mixing $d \leftrightarrow D$ $\mathcal{U}_L$-spin $\qquad \hat{d} \leftrightarrow \hat{D}$ right index $\mathcal{U}_R$-spin

$$\langle H(27)\rangle = \mathrm{Diag}\left(e^1_1\ ,\ e^2_2\ ,\ e^3_3\right) \quad \text{chaice of basis} : e^2_3 = e^3_2 = 0$$

$$\mathbf{27} \times \mathbf{27} = \overline{\mathbf{27}} + \overline{\mathbf{351}}_A + \overline{\mathbf{351}}_S$$

$$H = H(27) \ , \quad H_A = H(351_A) \ , \quad H_S = H(351_S)$$

$$\mathcal{L}_Y = \left((\Psi_r^\alpha)^T \mathrm{i}\sigma_2 \Psi_s^\beta \right) [G_{\alpha\beta} H_{rs} + A_{\alpha\beta}(H_A)_{rs} \\ S_{\alpha\beta}(H_S)_{rs}] + \mathrm{h.c.}$$

$$H \rightarrow (\bar{3}, 3, 1) \ ,$$
$$H_A \rightarrow (\bar{3}, 3, 1) + (\bar{3}, \bar{6}, 1) + (6, 3, 1)$$
$$H_S \rightarrow (\bar{3}, 3, 1) + (6, \bar{6}, 1) \ .$$

$$G\Psi\Psi H \rightarrow G\left(u\hat{u}e^1{}_1 + d\hat{d}e^2{}_2 - e\hat{e}e^2{}_2 - \nu\hat{\nu}e^1{}_1 + \ldots\right)$$

Dirac couplings

$$A\Psi\Psi H_A \rightarrow$$

$$S\Psi\Psi H_S \rightarrow S\,LLH_S(6,\overline{6},1) \rightarrow S\left(\hat{\nu}\hat{\nu}F^{\{2,2\}} + NN\,F^{\{3,3\}} + \hat{\nu}N\,F^{\{2,3\}}\ldots\right)$$

Needed for Majorana masses

- **G & A can be fixed from quark/lepton masses & mixings (some predictions).**
  S-matrix is free in general…
- **Aim: Relate S with G & A → more predictions**
- **Way: Avoid Hs & derive S_{eff}.**
  Possible by generation symmetry $SO(3)_g$

Generation (flavor) symmetry $SO(3)_g \times P_g$

Chiral

- Promote G & A to the field operators

$$G \to \frac{\chi}{M} \qquad\qquad A \to \frac{\xi}{M'}$$

$$SO(3)g: \qquad \psi \sim 3, \qquad \chi \sim 1+5, \qquad \xi \sim 3$$

- Do not introduce Hs(351).

`G' & `A' operators :

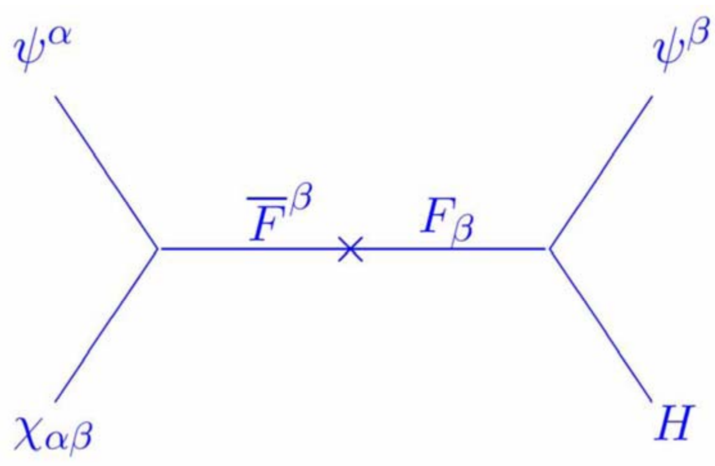
$$\frac{\chi}{M}\psi\,\psi\,H(27) + \frac{\xi}{M'}\psi\,\psi\,H_A(351)$$

- If no other flavor structure present →

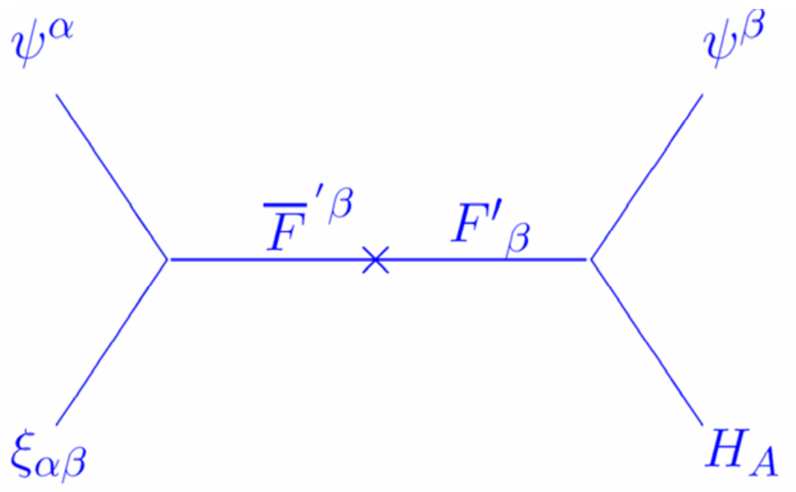
All derived matrices, including S, in effective theory will be functions of G & A (χ & ξ)

Great reduction of parameters

Effective operators through renorm. Terms by integrating massive spinors



$$\Longrightarrow \quad \frac{\langle \chi_{\alpha\beta} \rangle}{M} \left(\psi^\alpha H(27) \psi^\beta \right)$$



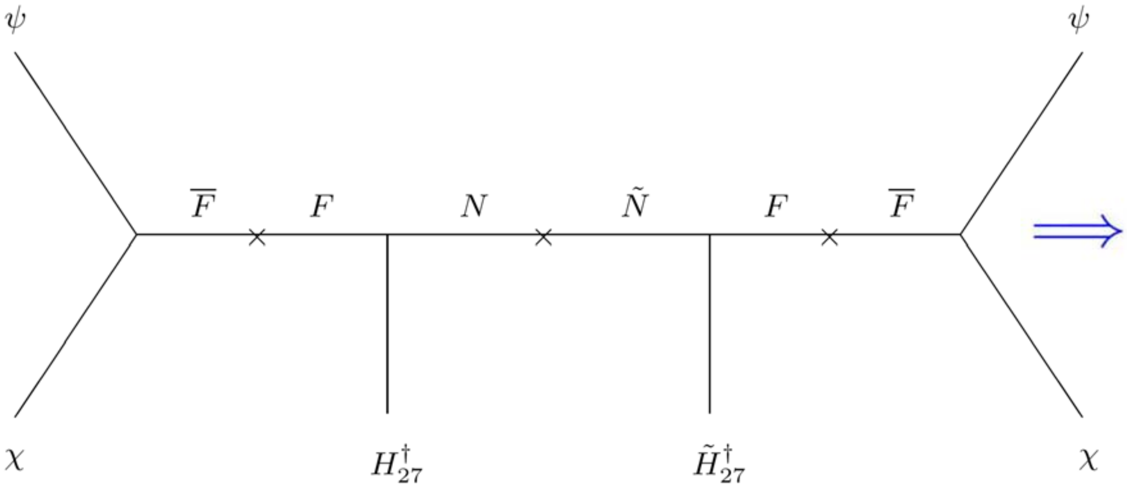
$$\Longrightarrow \quad \mathrm{i} \frac{\langle \xi_{\alpha\beta} \rangle}{M'} \left(\psi^\alpha H_A(351) \psi^\beta \right)$$

These induce masses of all fermions except $L^3{}_2 = \hat{\nu}$ *and* $L^3{}_3$

$L^3{}_2\, L^3{}_3 \sim (\overline{6},6,1)$ not contained in $H(27)$

However $H^+ \times H^+ \supset (6,\overline{6},1)$

Generate this by decoupling of heavy E6 singlets: $N(1,3)\,,\ \overline{N}(1,3)$



$$\Longrightarrow \frac{\langle \chi_{\alpha\beta} \rangle}{M} \frac{1}{M_N} \left(\psi^{\alpha} H^{\dagger} \tilde{H}^{\dagger} \psi^{\gamma} \right) \frac{\langle \chi_{\gamma\beta} \rangle}{M}$$

$L_2^3\,,\ L_3^3$ **'s masses can be generated now**

Effective Yukawa Couplings

- $$\mathcal{L}_Y^{\text{eff}} = G_{\alpha\beta}\left(\psi^\alpha H \psi^\beta\right) + A_{\alpha\beta}\left(\psi^\alpha H_A \psi^\beta\right) + \frac{1}{M_N}(G^2)_{\alpha\beta}\left(\psi^\alpha H^\dagger \tilde{H}^\dagger \psi^\beta\right)$$

Thanks to $E_6 x SO(3)_g$ the Generation structure is fixed

$$m_U \propto G = (\sigma^4, \sigma^2, 1) \qquad \sigma \simeq 0.05$$

$$A_{\alpha\beta} = \mathrm{i}\begin{pmatrix} 0 & \sigma & -\sigma \\ -\sigma & 0 & \frac{1}{2} \\ \sigma & -\frac{1}{2} & 0 \end{pmatrix}$$

From CKM fit **(B.S, Z.T'03)**

$$M(L_2^3, L_3^3) \propto G^2 \simeq (\sigma^8, \sigma^4, 1)$$

Superstrong hierarchy

Mass Matrices

- **Down Quarks**

$$M_{d,D} = \begin{array}{c} \\ d \\ D \end{array} \begin{array}{c} \begin{array}{cc} \hat{d} & \hat{D} \end{array} \\ \begin{pmatrix} m_b^0 G + h_2^2 A\,, & h_3^2 A \\ h_2^3 A\,, & e_3^3 G \end{pmatrix} \end{array}$$

- **Charged Leptons**

$$M_{e,E} = \begin{array}{c} e^- \\ E^- \end{array} \begin{pmatrix} -m_\tau^0 G - g_2^2 A\,, & g_2^3 A \\ -g_3^2 A\,, & -e_3^3 G \end{pmatrix}$$

6x6 To 3x3 reduction

$$M_d = m_b^0 + h_2^2 A - h\sigma^3 A G^{-1} A$$

$$M_e = -m_\tau^0 - g_2^2 A - g_0 \sigma^3 A G^{-1} A$$

(4 parameters vs 7 observables)

(3 params. → 3 mass, 3 angles)

* $h_2^2\ ,\ h_3^2\ ,\ h_2^3$ VEVs $\subset \langle H_A \rangle \sim (\overline{3}, 3, 1)$;

** $g_2^2\ ,\ g_3^2\ ,\ g_2^3$ VEVs $\subset \langle H_A \rangle \sim (\overline{3}, 6, 1)$

• Neutral Leptons → neutrino matrix

$$
M_L = \begin{array}{c} \\ L^2_3 \\ L^3_2 \\ L^3_3 \\ L^1_1 \\ L^2_2 \end{array}
\begin{array}{c} \begin{array}{ccccc} L^2_3 & L^3_2 & L^3_3 & L^1_1 & L^2_2 \end{array} \\
\begin{pmatrix}
0 & -e^1_1 G & 0 & -\, g^3_2 A & 0 \\
-e^1_1 G & 0 & M_1 & 0 & 0 \\
0 & M_1^T & M_2 & 0 & e^1_1 G \\
-g^3_2 A^T & 0 & 0 & 0 & M_0 \\
0 & 0 & e^1_1 G & M_0^T & 0
\end{pmatrix} \end{array}
$$

$$
M_0 = e^3_3 G \,, \qquad M_1 = F^{\{2,3\}} G^2 + F_A A \,, \qquad M_2 = F^{\{3,3\}} G^2
$$

Reduction by `multiple' see-saw: 15x15 to 3x3

$$
m_\nu \simeq -\frac{(e^1_1)^2}{(F^{\{2,3\}})^2} \left(F^{\{3,3\}} \mathbf{1} + F^{\{2,3\}} \frac{g^3_2}{e^3_3} (A \frac{1}{G} + \frac{1}{G} A^T) \right)
$$

$$
+ \text{corrections}(F_A, x_g^2)
$$

Composed by known G & A

$\Longrightarrow$ bimaximal neutrino mixing + important corrections

Results: Neutrinos

- **i)** Inverted
  hierarchy: $m_{\nu 1,2} \approx 0.06$ eV, $m_{\nu 3} \approx 0.037$ eV

- **ii)** $F^{\{2,3\}} \approx 2 \cdot 10^{13}$ GeV $\rightarrow$ $\Delta m^2_{\text{atm}} \simeq 0.0024\ (\text{eV})^2$
  suggests $e^3_3 \simeq M_I \simeq F^{\{2,3\}}$ $g_1(M_I) = g_2(M_I)$ $(SU(3))^3$

- Heavy fermion masses are fixed

  L^3_2, L^3_3- pair masses: $\approx (700\ ,\ 10^8\ ,\ 10^{13})$ GeV.

- $\langle m_{\beta\beta} \rangle \simeq 0.046$ eV

- $\theta_{12} \simeq 34^o$, $\theta_{23} \simeq 43^o$, $\theta_{13} \simeq 6.3^o$, $\delta_l \simeq 67^o$

Results: Charged leptons

$$\text{Fit mass} \quad m_\tau \;, \quad g_2^2 = 0.166 \text{ GeV} \qquad g_0 = 1.87 \text{ GeV}$$

$$m_\tau = 1.75 \text{ GeV} \quad m_\mu = 103 \text{ MeV} \quad m_e = 0.44 \text{ MeV}$$

$$|V_{ij}^{\text{ch}}| - \text{small}$$

Results: Quarks

$$\text{Fit mass } m_b \;, \quad h_2^2 \leq -0.214 \text{ GeV} \qquad h = 0.97 \text{ GeV}$$

- Masses: $m_b = 2.89$ GeV $\quad m_s = 50$ MeV $\quad m_d = 2.6$ MeV
- CKM Angles:

$$|V_{us}| = 0.228 \qquad |V_{cb}| = 0.042 \qquad |V_{ub}| = 0.0039$$

$$\alpha_q = 97^o \qquad \beta_q = 23^o \qquad \gamma_q = 60^o$$

- Lightest D quark: $m_{D_1} \approx 10^5$ TeV

Summary

With E_6 GUT & $SO(3)_g$ we obtained:

- Close connection between Quarks, Ch. Leptons and neutrinos;

- Model determines all (GUT) heavy fermion masses

$$M(\nu^c)\ ,\ M(D)\ ,\ M(L)\quad \subset 27 \text{ of } E_6$$

- Model Predicts:

Inverted hierarchical neutrinos;

$$\theta_{13} \simeq 6^o\ ,\quad \delta_l \simeq 67^0,\quad \langle m_{\beta\beta} \rangle \simeq 0.046 \text{ eV}$$

} **Can be tested In future !**

Thank You