KILLING GHOSTS

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TALK SUMMARY:

If theory analytic (tausal)
then quantization
SANS GHOSTS
is possible

LONG STANDING TABOO

"high derivative theories are sick"

$$\mathcal{L}(\varphi, \partial\varphi, \cancel{\partial^2\varphi, \partial^3\varphi}, \dots)$$

$$\Downarrow$$

GHOSTS.

POLES of NEGATIVE RESIDUE

Except for MAN-MADE ghosts
(Feynman, Faddeev Popov ...)

(NOT OUR SUBJECT)

GHOSTS INHIBIT PHYSICS

$$\mathcal{L} = -\frac{1}{4}\mathrm{tr}(F_{\mu\nu}F^{\mu\nu}) + \frac{1}{M^2}\mathrm{tr}\cancel{(FFF)}$$

Rebuild All from the Beginning

the ghost problem

$$\partial_\mu \partial^\mu \partial_\nu \partial^\nu \phi = J$$

$$\partial_\mu \rightarrow (i\omega, -i\vec{k})$$

$$(\omega^2 - \vec{k}^2)(\omega^2 - \vec{k}^2)\,\phi_{k\omega} = J_{k\omega}$$

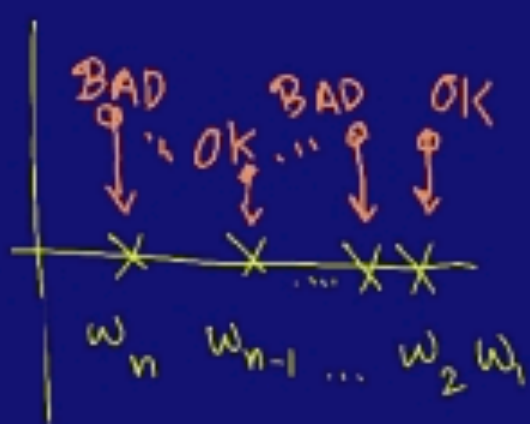
$$\phi_{k\omega} = \frac{J_{k\omega}}{(\omega^2 - k^2)(\omega^2 - k^2)}$$

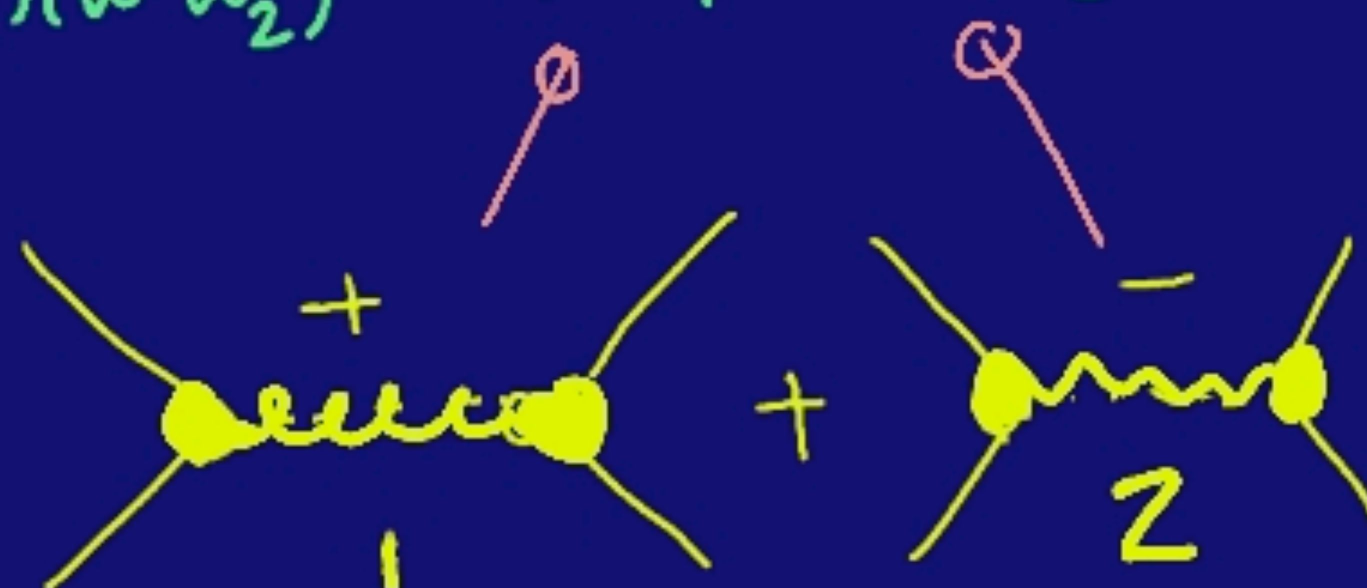
set $\omega = |\vec{k}|$

blows up!

ghost $\equiv$ pole with negative residue

$$\frac{1}{(\omega-\omega_1)(\omega-\omega_2)} = \left[\frac{1}{\omega-\omega_1} - \frac{1}{\omega-\omega_2}\right]\frac{1}{\omega_1-\omega_2}$$





negative probability

Julia's New Idea

... go back to $\boxed{1}$

$$\boxed{x^{(n)} = \partial^n x}$$

$$[\partial^n + a_{n-1}\partial^{n-1} + \dots a_1 \partial + a_0] x = 0$$

$$\frac{d}{dt}\begin{pmatrix} x \\ x^{(1)} \\ \vdots \\ x^{(n-1)} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & \dots \\ 0 & 0 & 1 & 0 & \dots \\ \dots & & & \dots & 0 \; 1 \\ -a_{n-1} & -a_{n-2} & \dots & & -a_0 \end{pmatrix}\begin{pmatrix} x \\ x^{(1)} \\ \vdots \\ x^{(n-1)} \end{pmatrix}$$

when are such equations Hamiltonian?

($q_1, q_2 \ldots$ WHICH ARE WHICH? $\ldots p_1, p_2 \ldots$)

similarity transforms $A \sim B \rightleftarrows$ $A = MSM^{-1}$

$$\mathbb{X} = \begin{pmatrix} x \\ x^{(1)} \\ x^{(2)} \\ \vdots \\ x^{(n-1)} \end{pmatrix} \sim \begin{pmatrix} q_1 \\ \vdots \\ q_m \\ p_1 \\ \vdots \\ p_m \end{pmatrix}$$

ostrogradski

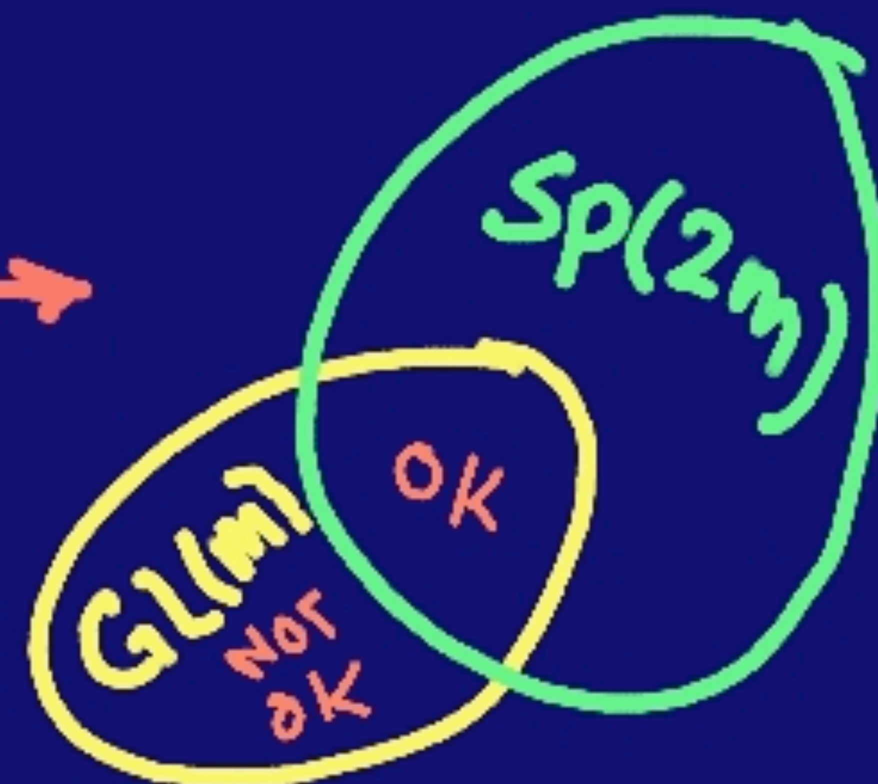
$q_1 \to x$

$q_2 \to x^{(1)}$

$q_3 \to x^{(2)}$

$\vdots$

$\rightarrow$



example mix-up:

$\cancel{Sp}$ $\quad q \to p$, $\quad p \to q$ $\qquad \dot{p} = \partial H/\partial q$, $\quad \dot{q} = -\partial H/\partial p$ $\quad$ NO!

Generators

$$S \in Sp(2m) \sim 1 + JH$$

$$J = \begin{pmatrix} 0 & 1_{m\times m} \\ -1_{m\times m} & 0 \end{pmatrix} \qquad H = H^T = \begin{pmatrix} h_1 & M \\ M^T & h_2 \end{pmatrix}$$

$$\Phi = \begin{pmatrix} q \\ p \end{pmatrix} \qquad S J S^T = J$$

$$\Phi_1 \cdot J \cdot \Phi_2 \to \Phi_1 S^T J S \Phi_2 = \Phi_1 \cdot J \cdot \Phi_2$$

$$\frac{d}{dt}\begin{pmatrix} x \\ x^{(1)} \\ \vdots \\ x^{(n-1)} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 & \cdots \\ 0 & 0 & 1 & 0 & \cdots \\ \cdots & & & \cdots & 0\,1 \\ -a_{n-1} & -a_{n-2} & \cdots & & -a_0 \end{pmatrix}\begin{pmatrix} x \\ x^{(1)} \\ \vdots \\ x^{(n-1)} \end{pmatrix}$$

Hamiltonian

$$\dot{X} = AX \qquad A \sim JH$$

UNITARITY

$$\frac{d}{dt}\langle\psi|\cdot\mathbb{1}\cdot|\psi\rangle=0$$

$$|\psi\rangle\to S|\psi\rangle$$

$$\langle\psi|\psi\rangle\to\langle\psi|S^{\dagger}S|\psi\rangle$$

"PT QUANTUM MECHANICS" (Bender)

$$\frac{d}{dt}(x\cdot g\cdot x)=\dot{x}\cdot g\cdot x+x\cdot g\cdot\dot{x}$$

$$=x(A^{T}g+gA)x\to 0$$

$$A^{T}g+gA\sim 0$$

$$g^{1/2}[g^{-1/2}A^{T}g^{1/2}+g^{-1/2}Ag^{1/2}]g^{1/2}=0$$

$$A\sim -A^{T}$$

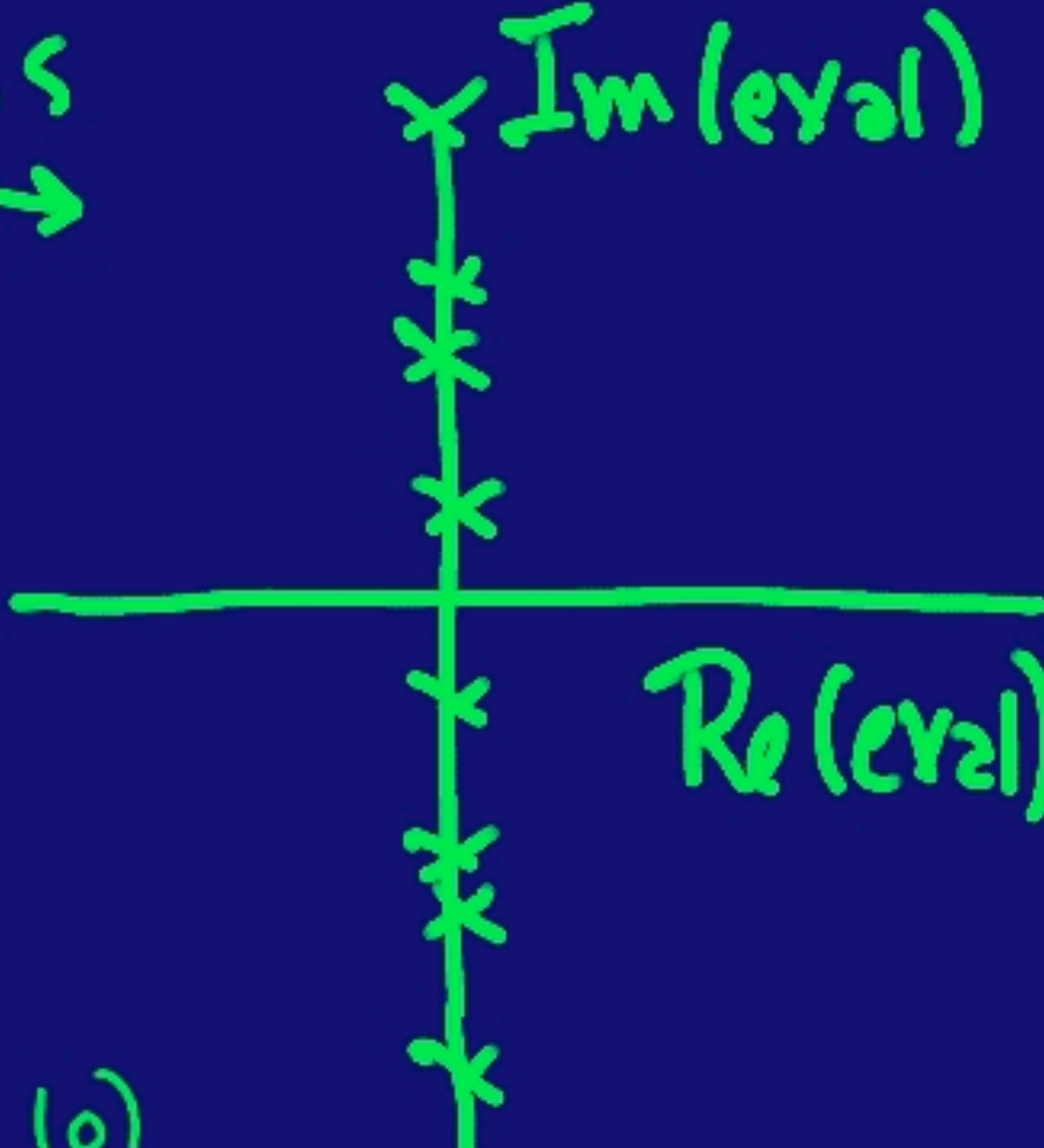
ok! since $A \sim -A^T$

$$A \sim \begin{pmatrix} -i\omega_1 & & & & & \\ & \ddots & & & & \\ & & -i\omega_m & & & \\ & & & i\omega_1 & & \\ & & & & \ddots & \\ & & & & & i\omega_m \end{pmatrix}$$

$$A e_\alpha = -i\omega_\alpha e_\alpha$$

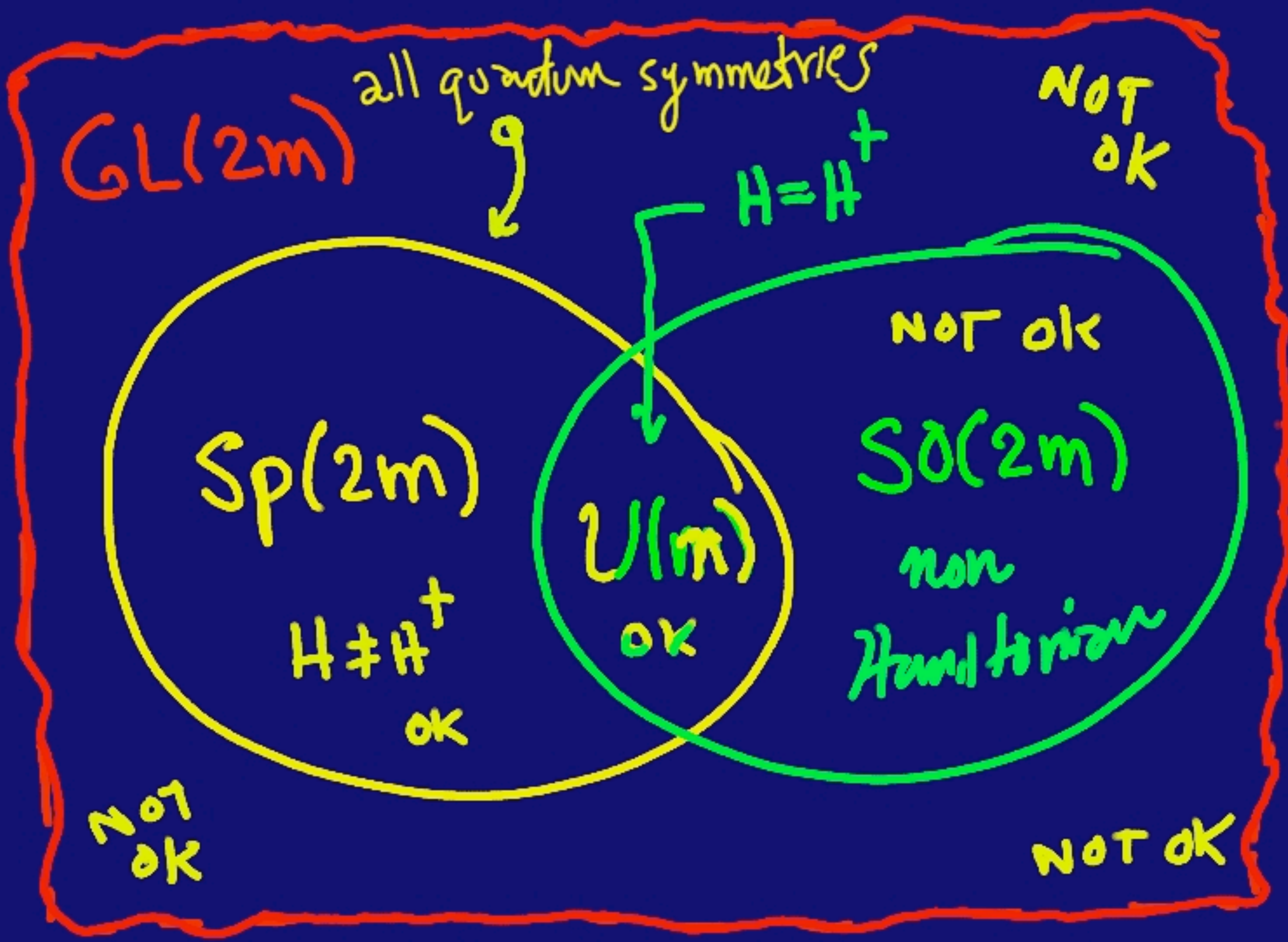
$$e_\alpha(\omega_\alpha) = e_\alpha^*(-\omega_\alpha)$$

evals



$$e_\alpha(t) \sim e^{-i\omega t} e_\alpha(0)$$

$\Longleftrightarrow$ $\boxed{q_\alpha = \mathrm{Re}\, e_\alpha \qquad p_\alpha = \mathrm{Im}\, e_\alpha}$



GL(2m)
all quantum symmetries
g
H=H†
NOT ok
Sp(2m)
H≠H†
ok
U(m)
ok
NOT ok
SO(2m)
non Hamiltonian
NOT ok
NOT OK

"2 state model"

$$L = \frac{1}{2}\ddot{x}^2 - (\omega_1^2 + \omega_2^2)\dot{x}^2 + \omega_1^2\omega_2^2 x$$

$$x^{(4)} + (\omega_1^2 + \omega_2^2)\ddot{x} + \omega_1^2\omega_2^2 x = 0$$

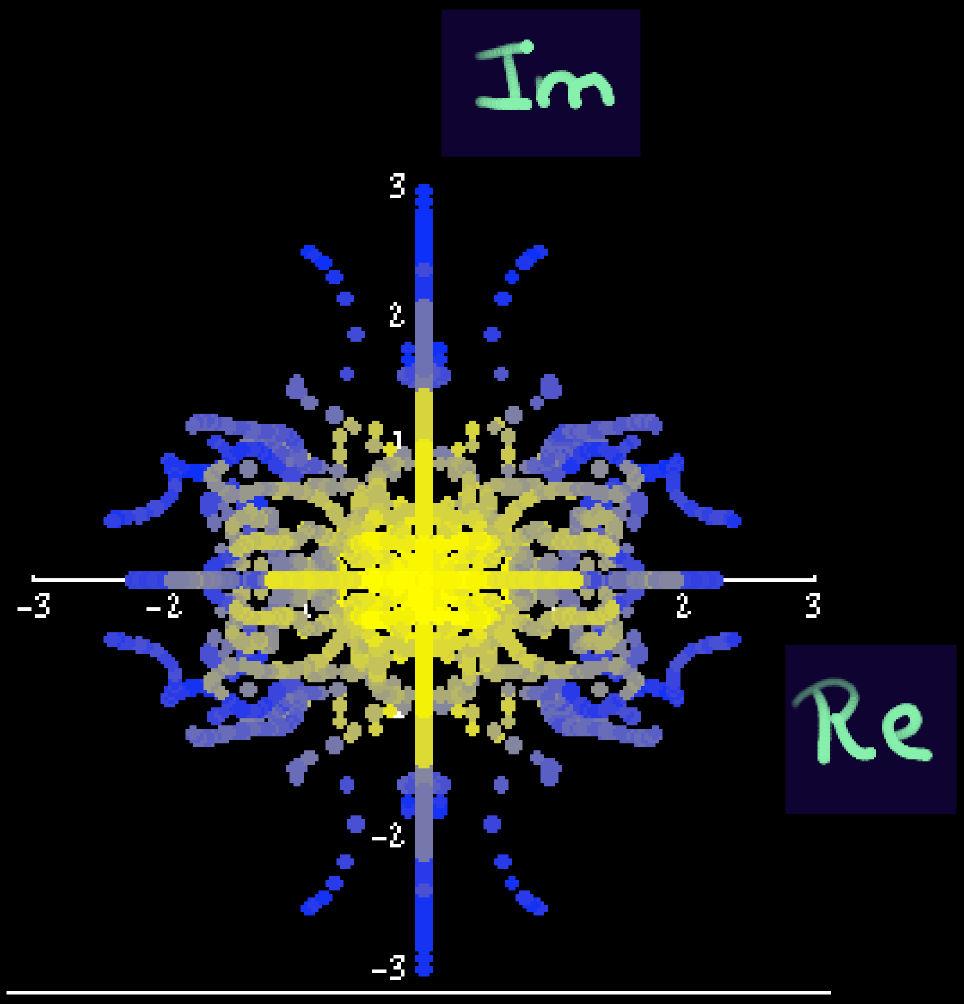
$\dot{x} = \Lambda X \to$ diagonal $\to q, p$ ... solve

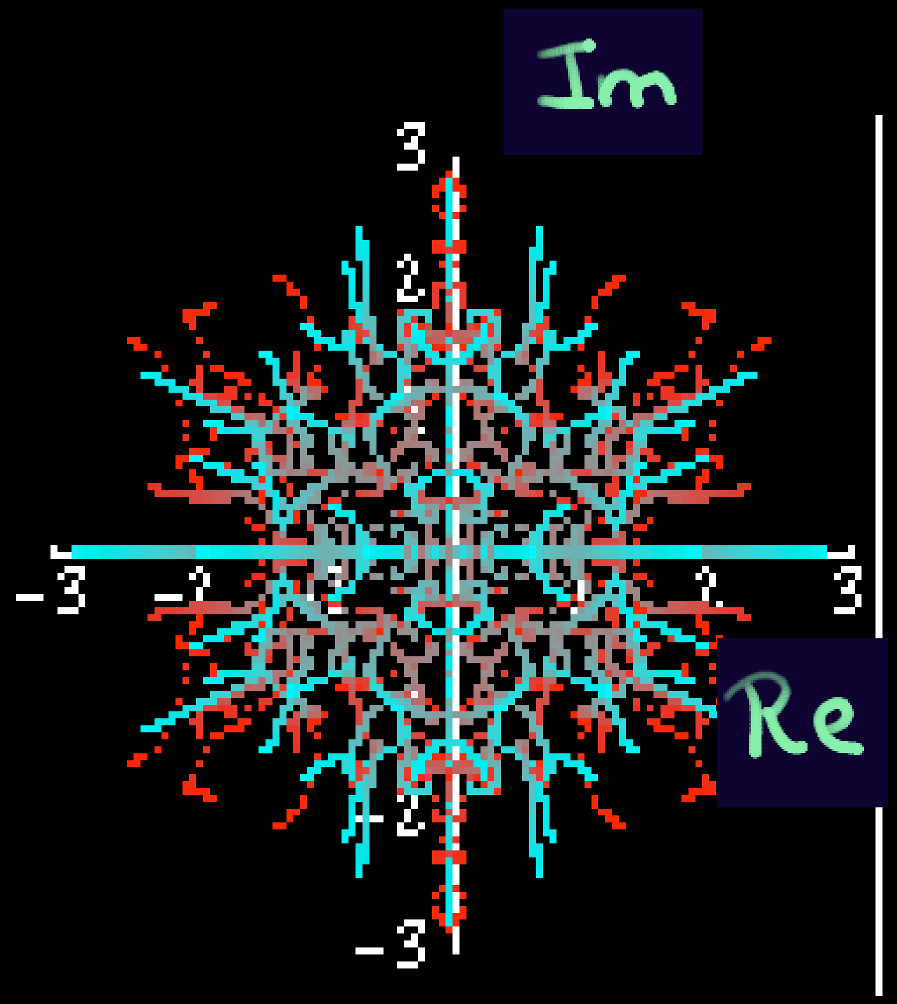
$$\begin{pmatrix} x \\ \dot{x} \\ \ddot{x} \\ \dddot{x} \end{pmatrix} = \begin{pmatrix} 1 & -\omega_1^2 & 0 & 0 \\ 0 & 0 & \omega_1 & -\omega_1^3 \\ 1 & -\omega_2^2 & 0 & 0 \\ 0 & 0 & \omega_2 & -\omega_2^3 \end{pmatrix} \begin{pmatrix} q_1 \\ q_2 \\ p_1 \\ p_2 \end{pmatrix}$$

$$H = \frac{p_1^2}{2} + \frac{\omega_1^2}{2} q_1^2 + \frac{p_2^2}{2} + \frac{\omega_2^2}{2} q_2^2$$

$$E_{n_1, n_2} = (n_1 + 1/2)\omega_1 + (n_2 + 1/2)\omega_2$$

trajectories of eigenvalues





SYMPLECTIC PARITY

$$q \to p \qquad \cancel{Sp} \quad \cancel{U} \quad SO(2m)$$
$$p \to q$$

$$a = q + ip \to p + iq$$

norms are reversed

$$\langle 0 | a a^{\dagger} | 0 \rangle = -1$$

ghosts caused by Sp, parity mixup!

Summary

- $\partial^n \phi \rightarrow n$ generalized coordinates
- q & p are inequivalent.
  Go find them!
- Can always avoid a ghost.

GHOSTS DO NOT EXIST

DOOR TO NEW THEORIES IS OPEN