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Anomaly-free discrete family symmetries

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Tri-bimaximal mixing

- leptonic mixing measured in neutrino oscillation experiments
- two large and one small angle

MNSP-angles	3σ exp.	tri-bimax.
$\sin^2\theta_{12}$:	$0.24 - 0.40$	0.33
$\sin^2\theta_{23}$:	$0.34 - 0.68$	0.50
$\sin^2\theta_{13}$:	≤ 0.041	0

$$U_{\mathrm{MNSP}} \approx U_{\mathcal{TB}} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

$\Longrightarrow$ structure might originate in non-Abelian discrete symmetry $\mathcal{G}$

Possible discrete symmetries

- symmetry group: SM $\times\, \mathcal{G}$
- three families
- $\mathcal{G}$ should have two- or three-dimensional irreps
- $\mathcal{G}$ is a finite subgroup of either
  - $SU(3)$ e.g. $\mathcal{PSL}_2(7)$, $\mathcal{Z}_7 \rtimes \mathcal{Z}_3$, $\Delta(27)$, $\mathcal{S}_4$, $\mathcal{A}_4$, $\mathcal{D}_5$, $\mathcal{S}_3$
  - $SO(3)$ e.g. $\mathcal{S}_4$, $\mathcal{A}_4$, $\mathcal{D}_5$, $\mathcal{S}_3$
  - $SU(2)$ e.g. $\mathcal{T}'$

example: $\mathcal{A}_4$ (irreps $\mathbf{1}$, $\mathbf{1}'$, $\overline{\mathbf{1}'}$, $\mathbf{3}$)

$$L \sim \mathbf{3}\,, \qquad E^c \sim \mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'}\,, \qquad N^c \sim \mathbf{3}$$

Gauging discrete symmetries

- discrete symmetries violated by quantum gravity effects
- unless they are remnants of a gauge symmetry

$\longrightarrow$ "discrete gauge symmetry"

$$\boxed{G_f \supset \mathcal{G}}$$

gauge symmetry discrete symmetry

A N O M A L I E S

Potential anomalies

$$SU(3)_C \times SU(2)_W \times U(1)_Y \times G_f$$

- $G_f = U(1)$

$SU(3)_C - SU(3)_C - U(1)$ $\qquad$ $U(1)_Y - U(1)_Y - U(1)$

$SU(2)_W - SU(2)_W - U(1)$ $\qquad$ $U(1)_Y - U(1) - U(1)$

$\text{Gravity} - \text{Gravity} - U(1)$ $\qquad$ $U(1) - U(1) - U(1)$

$\longrightarrow$ constraints on possible $\mathcal{Z}_N \subset U(1)$ symmetries

- $G_f = SU(3)$

$SU(3) - SU(3) - SU(3)$ $\qquad$ $SU(3) - SU(3) - U(1)_Y$

$\longrightarrow$ What can we extract in this case?

Irreps $\boldsymbol{\rho}$ of G_f and their indices

cubic index: $$\mathrm{Tr}\left(\left\{T_a^{[\boldsymbol{\rho}]}\,,\,T_b^{[\boldsymbol{\rho}]}\right\}\,T_c^{[\boldsymbol{\rho}]}\right) \;=\; A(\boldsymbol{\rho})\,\frac{d_{abc}}{2}$$

quadratic index: $$\mathrm{Tr}\left(\left\{T_a^{[\boldsymbol{\rho}]}\,,\,T_b^{[\boldsymbol{\rho}]}\right\}\right) \;=\; \ell(\boldsymbol{\rho})\,\delta_{ab}$$

Irreps $\boldsymbol{\rho}$ of $SU(3)$	$\ell(\boldsymbol{\rho})$	$A(\boldsymbol{\rho})$
$(10):\ \mathbf{3}$	1	1
$(20):\ \mathbf{6}$	5	7
$(11):\ \mathbf{8}$	6	0
$(30):\ \mathbf{10}$	15	27
$(21):\ \mathbf{15}$	20	14

Irreps $\boldsymbol{\rho}$ of $SO(3)$	$\ell(\boldsymbol{\rho})$
$\mathbf{3}$	1
$\mathbf{5}$	5
$\mathbf{7}$	14
$\mathbf{9}$	30
$\mathbf{11}$	55

anomaly freedom:

$$\sum_{\boldsymbol{\rho}} A(\boldsymbol{\rho}) \;=\; 0 \qquad\qquad \sum_{\boldsymbol{\rho}} \ell(\boldsymbol{\rho})\cdot Y(\boldsymbol{\rho}) \;=\; 0$$

Irreps $\mathrm{r_i}$ of $\mathcal{G}$ and their indices

- use of irreps $\mathbf{r_i}$ of $\mathcal{G}$ in flavor model building
- $\mathbf{r_i}$ can originate from different irreps $\boldsymbol{\rho}$ of G_f

$$\boldsymbol{\rho} \longrightarrow \sum_i a_i \, \mathbf{r_i} \qquad\qquad (a_i = \text{multiplicities})$$

- define "discrete indices" $\widetilde{A}(\mathbf{r_i})$, $\widetilde{\ell}(\mathbf{r_i})$ such that:

$$\begin{aligned} A(\boldsymbol{\rho}) &= \sum_i a_i \, \widetilde{A}(\mathbf{r_i}) \mod N_A \\ \ell(\boldsymbol{\rho}) &= \sum_i a_i \, \widetilde{\ell}(\mathbf{r_i}) \mod N_\ell \end{aligned}$$

Embedding $\mathcal{A}_4$ into $SU(3)$

discrete indices of $\mathcal{A}_4$:

Irreps $\mathbf{r_i}$ of $\mathcal{A}_4$	$\widetilde{\ell}(\mathbf{r_i})$ $(N_\ell = 12)$	$\widetilde{A}(\mathbf{r_i})$ $(N_A = 2)$
$\mathbf{1}$	0	0
$\mathbf{1}'$	2	0
$\overline{\mathbf{1}'}$	2	0
$\mathbf{3}$	1	1

$\boldsymbol{\rho}$	$\sum_i a_i\, \mathbf{r_i}$	$\ell(\boldsymbol{\rho})$	$\sum_i a_i\, \widetilde{\ell}(\mathbf{r_i})$	$A(\boldsymbol{\rho})$	$\sum_i a_i\, \widetilde{A}(\mathbf{r_i})$
$\mathbf{3}$	$\mathbf{3}$	1	1	1	1
$\mathbf{6}$	$\mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'} + \mathbf{3}$	5	5	7	1
$\mathbf{8}$	$\mathbf{1}' + \overline{\mathbf{1}'} + 2 \cdot \mathbf{3}$	6	6	0	2
$\mathbf{10}$	$\mathbf{1} + 3 \cdot \mathbf{3}$	15	3	27	3
$\mathbf{15}$	$\mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'} + 4 \cdot \mathbf{3}$	20	8	14	4

Embedding $\mathcal{A}_4$ into $SO(3)$

discrete indices of $\mathcal{A}_4$:

Irreps $\mathbf{r_i}$ of $\mathcal{A}_4$	$\widetilde{\ell}(\mathbf{r_i})$ $(N_\ell = 12)$	
$\mathbf{1}$	0	
$\mathbf{1}'$	2	
$\overline{\mathbf{1}'}$	2	
$\mathbf{3}$	1	

$\boldsymbol{\rho}$	$\sum_i a_i\,\mathbf{r_i}$	$\ell(\boldsymbol{\rho})$	$\sum_i a_i\,\widetilde{\ell}(\mathbf{r_i})$		
$\mathbf{3}$	$\mathbf{3}$	1	1		
$\mathbf{5}$	$\mathbf{1}' + \overline{\mathbf{1}'} + \mathbf{3}$	5	5		
$\mathbf{7}$	$\mathbf{1} + 2\cdot\mathbf{3}$	14	2		
$\mathbf{9}$	$\mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'} + 2\cdot\mathbf{3}$	30	6		
$\mathbf{11}$	$\mathbf{1}' + \overline{\mathbf{1}'} + 3\cdot\mathbf{3}$	55	7		

Discrete anomaly conditions

family symmetry G_f particles live in irreps $\boldsymbol{\rho}$

$$\sum_{\boldsymbol{\rho}} A(\boldsymbol{\rho}) = 0 \qquad \sum_{\boldsymbol{\rho}} \ell(\boldsymbol{\rho}) \cdot Y(\boldsymbol{\rho}) = 0$$

$\downarrow$

family symmetry $\mathcal{G}$ particles live in irreps $\mathbf{r_i}$

- some acquire masses (heavy)
- some don't (light)

$$\sum_{i=\text{light}} \widetilde{A}_i + \sum_{i=\text{heavy}} \widetilde{A}_i = 0 \bmod N_A$$

$$\sum_{i=\text{light}} \widetilde{\ell}_i Y_i + \sum_{i=\text{heavy}} \widetilde{\ell}_i Y_i = 0 \bmod N_\ell$$

An $\mathcal{A}_4$ example

– all hypercharges are integer

– hypercharge normalization $Y_Q = 1$

$$\left.\begin{array}{l}\\ \\ \end{array}\right\} \quad \begin{array}{|lcl|}\hline \sum_{i=\text{light}} \widetilde{A}_i & = & 0 \bmod 1 \\ \sum_{i=\text{light}} \widetilde{\ell}_i \, Y_i & = & 0 \bmod 12 \\ \hline \end{array}$$

- maybe $\mathcal{A}_4$ is a special feature of the lepton sector
- quarks transform as singlets under $\mathcal{A}_4$
- only leptons contribute to discrete anomaly

$$L \sim \mathbf{3}\,, \qquad E^c \sim \mathbf{1} + \mathbf{1}' + \overline{\mathbf{1}'}\,, \qquad N^c \sim \mathbf{3}$$

$$2 \times 1 \cdot (-3) \;+\; 4 \cdot 6 \;+\; 1 \cdot 0 \;=\; 18 \;\neq\; 0 \bmod 12$$