Karlsruhe Institute of Technology

Unitarization and Simplified Models

based on hep-ph/1511.00022 and hep-ph/1408.6207

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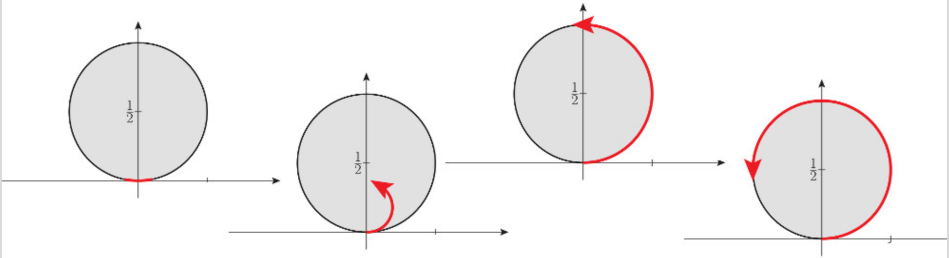
Institute of Theoretical Physics (ITP)



Outline

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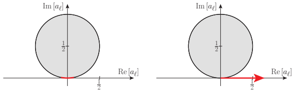
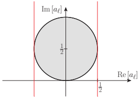
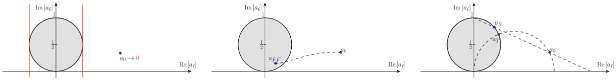
1. Introduction



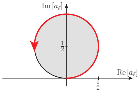
2. Unitarity



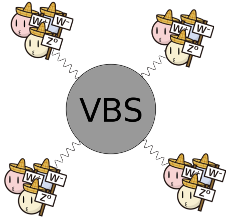
3. Unitarization



4. Simplified Models



VBS and the Standard Model



higgstan.com

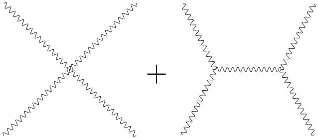
2013: Vector boson scattering is observed

$\Rightarrow$ The Higgs mechanism works as expected

Longitudinal vector boson self interaction

VBS amplitude rises with energy

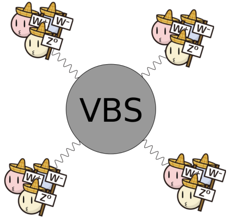
$\Rightarrow$ violation of unitarity



$$= \mathcal{O}\left(E^2\right)$$

$\mathcal{O}\left(E^4\right)$ $\mathcal{O}\left(E^4\right)$

VBS and the Standard Model



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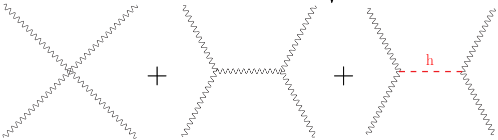
2013: Vector boson scattering is observed

$\Rightarrow$ The Higgs mechanism works as expected

VBS in the SM

Higgs exchange cancels the energy rise in VBS

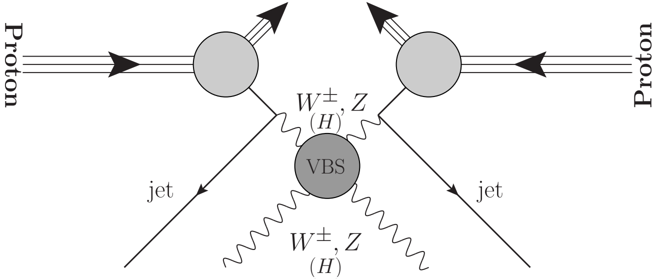
$\Rightarrow$ restores unitarity if $m_h \leq \sqrt{4\pi\sqrt{2}/G_F}$ Lee,Quig,Thacker 1977



$$+ \quad + \quad = \mathcal{O}\left(E^0\right)$$

$$\mathcal{O}\left(E^4\right) \qquad \mathcal{O}\left(E^4\right) \qquad \mathcal{O}\left(E^2\right)$$

VBS at the LHC



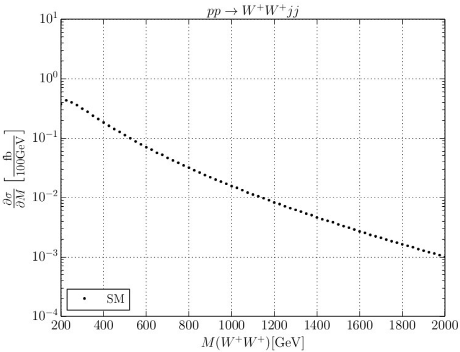
- VBS amplitude is bounded (weakly int.)

$\Rightarrow$ Cross section suppressed by PDF

- Look for deviation from the SM prediction

$\rightarrow$ Sensitive test of new physics contributions

VBS at the LHC



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$\Rightarrow$ Cross section suppressed by PDF

- Look for deviation from the SM prediction

$\rightarrow$ Sensitive test of new physics contributions

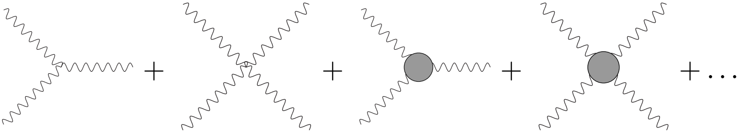
Modelling Physics beyond the SM

Desirable features of a generic SM extension

- Recovers the SM in an appropriate limit
- Respects established symmetries: $SU(3)_C \times SU(2)_L \times U(1)_Y$
- Captures any new physics
  (+ guidance where physics impact is large)
- Possibility to calculate radiative corrections

Effective Field Theory (EFT)

$$\mathcal{L} \quad = \quad \mathcal{L}_{SM} \quad + \quad \sum_{d\geq 4} \sum_i \frac{C_i}{\Lambda^{d-4}} \mathcal{O}_i^d$$



Longitudinal EFT Operators

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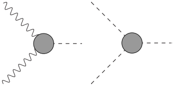
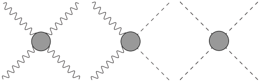
Anomalous couplings effecting longitudinal VBS

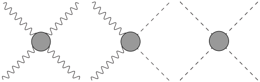
$$\mathcal{L}_{HD} = F_{HD}\ \mathrm{tr}\left[\mathbf{H}^\dagger \mathbf{H} - \frac{v^2}{4}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \left(\mathbf{D}^\mu \mathbf{H}\right)\right]$$



$$\mathcal{L}_{S,0} = F_{S,0}\ \mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}_\nu \mathbf{H}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}^\mu \mathbf{H}\right)^\dagger \mathbf{D}^\nu \mathbf{H}\right]$$



$$\mathcal{L}_{S,1} = F_{S,1}\ \mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}_\nu \mathbf{H}\right)^\dagger \mathbf{D}^\nu \mathbf{H}\right]$$

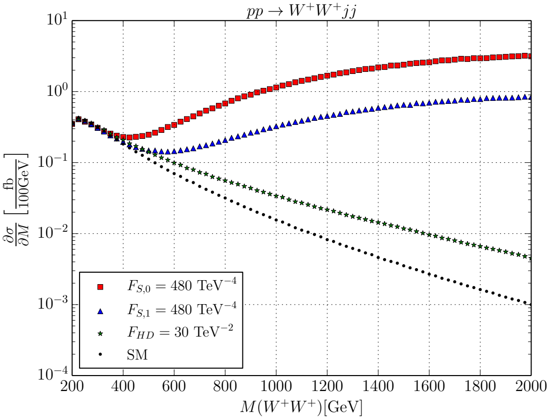


Linear Higgs matrix representation

$$\mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h - \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix}$$

$$F_{HD} \sim 1/\Lambda^2$$
$$F_{S,0} \sim 1/\Lambda^4$$
$$F_{S,1} \sim 1/\Lambda^4$$

Differential cross section at LHC (14 TeV)



- AQGC amplitudes rise with energy $\sim E^4$

$\rightarrow$ $D = 8$ Operators cancel the PDF suppression

- Unitarity obviously violated (at which energy?)

Unitarity

1. Unitarity of scattering matrix $S = \mathbb{1} + \mathrm{i}\mathbf{T} : \rightarrow \mathrm{i}\left(\mathbf{T} - \mathbf{T}^\dagger\right) = \mathbf{T}\mathbf{T}^\dagger$
2. Angular momentum conservation:
   conventionally normalized partial wave amplitudes a_ℓ
3. Unitarity implies

Argand-circle condition

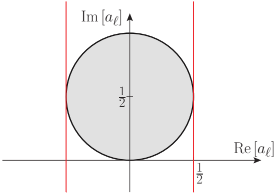
$$\left| a_\ell(s) - \frac{\mathrm{i}}{2} \right| \leq \frac{1}{2}$$

→ Outside: unitarity broken

→ Inside/On: unitarity fulfilled

inside: inelastic scattering ($<$)

on: elastic scattering ($=$)



Bound on real part

$$|\mathrm{Re}\,(a_\ell(s))| \leq \frac{1}{2}$$

Isospin-Spin Eigenamplitudes

- Weak boson interaction matrix has non-diagonal elements (GBET):

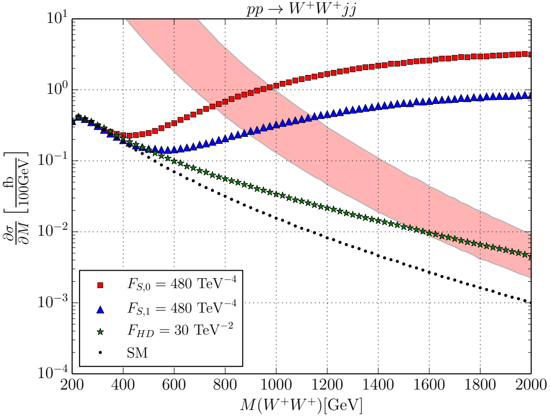
$$\begin{pmatrix} w^+w^+ \to w^+w^+ & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & w^+z \to w^+z & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w^+h \to w^+h & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & zh \to zh & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & w^+w^- \to w^+w^- & w^+w^- \to zz & w^+w^- \to hh \\ 0 & 0 & 0 & 0 & zz \to w^+w^- & zz \to zz & zz \to hh \\ 0 & 0 & 0 & 0 & hh \to w^+w^- & hh \to zz & hh \to hh \end{pmatrix}$$

⇒ Use isospin $SU_C(2)$ to diagonalize interaction matrix

- Partial wave decomposition into isospin-spin eigenamplitudes $a_{I\ell}$
- All $a_{I\ell}$ have to fulfill the Argand-circle condition
- Example for $a(W^+W^+ \to W^+W^+)$

$$a(w^+w^+ \to w^+w^+) = a_{20}(s) - 10a_{22}(s) - 15a_{22}(s)\frac{t^2+u^2}{s^2}$$

Unitarity bounds of Dim-8 Operators



- EFT for AQGC at current experimental bounds violates unitarity below 1 TeV (red band: variation of $a_{I\ell}$)

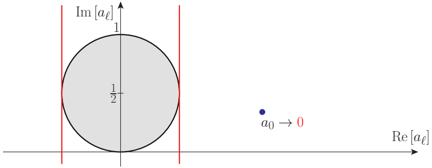
⇒ Naive EFT description is unphysical within LHC energy reach

- Extrapolation?

Cut-Off Method

- Cut-Off function: $\Theta\left(\Lambda_C^2 - s\right)$
- Λ_C equals unitarity bound (often: 0th partial wave)

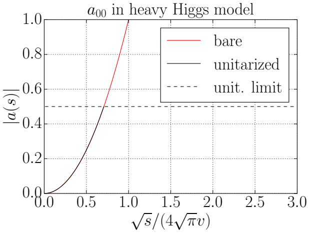
Ignoring high energy regime
beyond unitarity bound



- Requirement:
  experimental reconstruction of s

$\Rightarrow$ Difficult for *WW* final states

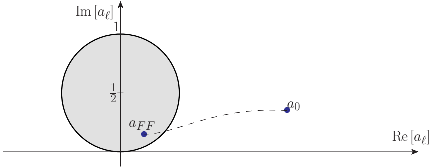
- Example: Higgs-less amplitude



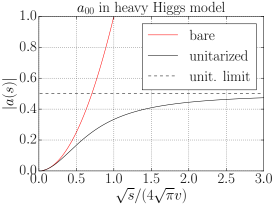
Dipole Form Factor

- Form-Factor: $\left(1 + \frac{s}{\Lambda_{FF}^2}\right)^{-p}$ Baur,Zeppenfeld 1988
- p is chosen accordingly to the EFT-operator dimension
- Λ_{FF} set to highest possible value that satisfy real unitarity bound (0th)

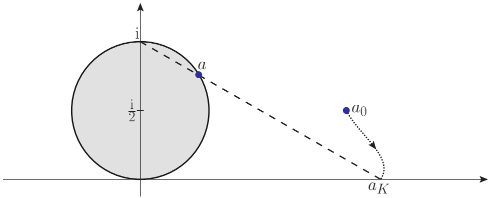
Projection inside the Argand circle



- Can be easily implemented for arbitrary anomalous operator
- Needs "Fine Tuning"
- Complete amplitude receives suppression factor
- Example: Higgs-less amplitude



K Matrix



Cayley Transform

Heitler, 1941

$S = \frac{\mathbf{1}+\mathrm{i}K/2}{\mathbf{1}-\mathrm{i}K/2}$,

where $K = K^\dagger$

and $S = \mathbf{1} + \mathrm{i}T$

Original K Matrix algorithm

Gupta, 1951/1981

- a_0: Compute T_0 matrix perturbatively
- a_K: Reconstruct K matrix order by order
- a: Insert into S matrix formula, without expanding again

  $a = \frac{a_k}{\mathbb{1} - \mathrm{i}a_k}$

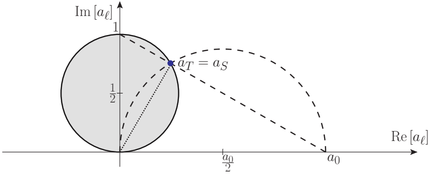
Relies on perturbation theory

$\Rightarrow$ Compute unitarized T matrix directly

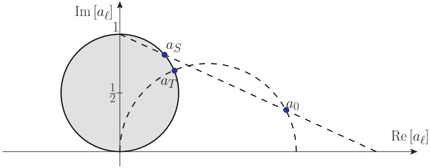
Direct T-Matrix Unitarization

1. Linear construction "Stereographic": $T = \dfrac{\mathrm{Re}\, T_0}{\mathbf{1} - \frac{\mathrm{i}}{2} T_0^\dagger}$
2. Circular construction "Thales": $T = \dfrac{1}{\mathrm{Re}\left(\frac{1}{T_0}\right) - \frac{\mathrm{i}}{2}\mathbf{1}}$

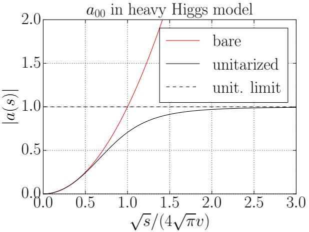
Start from real amplitude a_0:



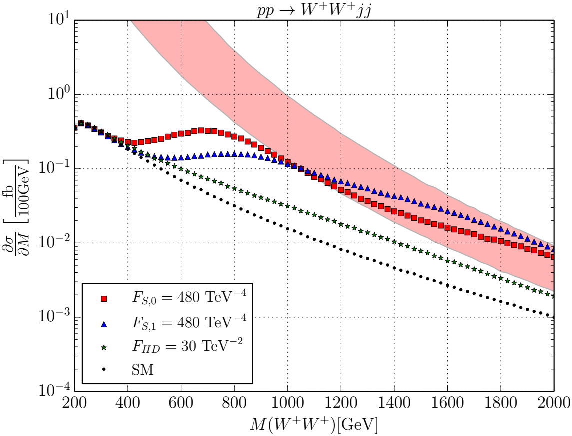
Start from complex amplitude a_0:



- Unitary amplitude left invariant
- But scheme dependence for complex a_0
- Example: Higgs-less amplitude



T-matrix Unitarization

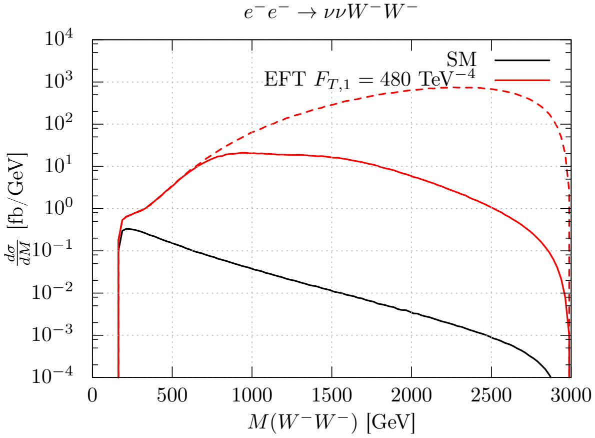


⇒ Saturation of isospin-spin amplitudes at their unitarity limit

- Leaves scattering matrices, which satisfy unitarity, invariant

! Introducing model dependence

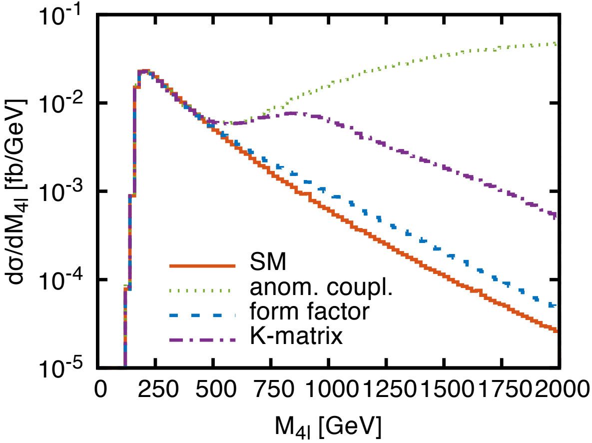
T-Matrix for transversal couplings



provided by C. Fleper(WHIZARD)

- Implementation of transversal couplings in validation
- Example: $\mathcal{L}_{T,1} = g^4 \,\mathrm{tr}\left[\mathbf{W}_{\alpha\mu}\mathbf{W}^{\mu\beta}\right] \mathrm{tr}\left[\mathbf{W}_{\beta\nu}\mathbf{W}^{\nu\alpha}\right]$

Comparison for $pp \to e^+\nu_e\mu^+\nu_\mu jj$



provided by M.Rauch (VBFNLO)

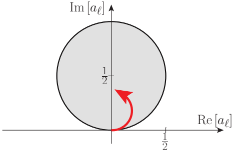
- EFT parameters: $F_{S,1} = 400\,TeV^{-4}$
- FF parameters: $p = 2,\ \Lambda_{FF} = 832\ \mathrm{GeV}$

Implementation in Matrix Element Generators

- MADGRAPH
  - Form-Factor
  - (Costumized unitarization method could be implemented through bias modules)
- VBFNLO
  - Form-Factor including tool to determine Λ_{FF}
  - K/T-matrix for $\mathcal{O}_{S,0}, \mathcal{O}_{S,1}$
- WHIZARD/OMEGA
  - K/T-matrix for $\mathcal{O}_{HD}, \mathcal{O}_{S,0}, \mathcal{O}_{S,1}, (\mathcal{O}_{T,0}, \mathcal{O}_{T,1}, \mathcal{O}_{T,2})$ and simplified models

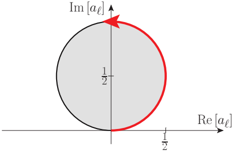
Scenarios for New Physics at High Energies

1. Inelastic



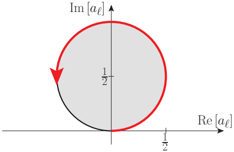
EFT+ Form-factor

2. Saturation



EFT elastic + T-Matrix

3. Resonance

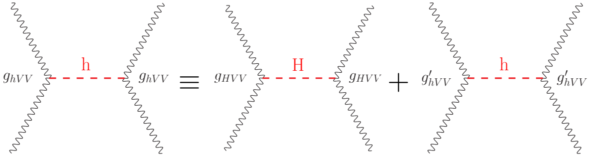


Adding additional resonances

The rise of an amplitude (AQGC) may be an expansion of a resonance

Adding additional heavy Higgs

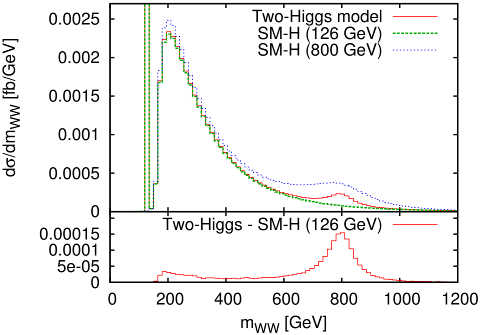
1. Adding additional heavy Higgs with mass m_H and coupling g_{HVV}
2. To satisfy unitarity: $g_{hVV} = g'_{hVV} + g_{HVV}$



3. Unitarity gives bounds to $m_H \leq m_H^B\left(g_{HVV}, m_h\right)$

- Modified coupling for light Higgs as compared to SM
- For generic resonances $\rightarrow$ Simplified models

Adding additional heavy Higgs



provided by D. Zeppenfeld (VBFNLO)

- Modified coupling for light Higgs as compared to SM
- For generic resonances → Simplified models

Guideline for Simplified Models

Introduction of generic resonances

- Use EFT-framework
- Introducing custodial $SU(2)_C$ symmetry $m_Z \approx m_W$
- Allow resonances in all accessible spin/isospin channels (here: only Higgs sector)
- Include extra anomalous couplings (reproduce unitary two Higgs model with $F_{HD} = -\frac{2}{v^2}\left(1 \pm \sqrt{\frac{v^2}{4}F_\sigma^2 + 1}\right)$)
- Beyond the resonance, the amplitude may eventually rise

$\Rightarrow$ Apply T-matrix unitarization scheme

Resonances in VBS: Quantum Numbers

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Spin

- Just consider Spin 0,2
- Spin 1 has different pheno (W/Z-mixing)

Symmetry

$$SU(2)_L \times SU(2)_R \to SU(2)_C$$

$$(\mathbf{0}, \mathbf{0}) \to 0$$

$$(\mathbf{1}, \mathbf{1}) \to 2 + 1 + 0$$

	isoscalar	isotensor
scalar	σ^0	$\begin{pmatrix} \phi_t^{--}, \phi_t^{-}, \phi_t^{0}, \phi_t^{+}, \phi_t^{++} \\ \phi_v^{-}, \phi_v^{0}, \phi_v^{+} \\ \phi_s^{0} \end{pmatrix}$
tensor	f^0	$\begin{pmatrix} X_t^{--}, X_t^{-}, X_t^{0}, X_t^{+}, X_t^{++} \\ X_v^{-}, X_v^{0}, X_v^{+} \\ X_s^{0} \end{pmatrix}$
...	...	...

Integrate out Isoscalar-scalar Resonance

- Simple example: Extension via scalar singlet σ:

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(m_\sigma^2 + \partial^2\right)\sigma + \sigma J_\sigma$$

$$J_\sigma = F_\sigma \,\mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right] \qquad \text{where} \quad F_\sigma \propto \frac{1}{\Lambda}$$

- Scalar mass is beyond experimental energy reach
- Integrate out heavy scalar resonance

$\Rightarrow$ Effective Lagrangian

$$\mathcal{L}_\sigma^{\mathrm{eff}} = \frac{F_\sigma^2}{2m_\sigma^2}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right]\mathrm{tr}\left[\left(\mathbf{D}_\nu \mathbf{H}\right)^\dagger \mathbf{D}^\nu \mathbf{H}\right]$$

- Leads to following AQGC

$$F_{S,1} = \frac{F_\sigma^2}{2m_\sigma^2}, \qquad \mathbf{H} = \frac{1}{2}\begin{pmatrix} v + h + \mathrm{i}w^3 & -\mathrm{i}\sqrt{2}w^+ \\ -\mathrm{i}\sqrt{2}w^- & v + h + \mathrm{i}w^3 \end{pmatrix}$$

Tensor: Fierz-Pauli

- Start with Fierz Pauli-Lagrangian for symmetric tensor

Fierz-Pauli Lagrangian

$$\mathcal{L}_{\mathrm{FP}} = \frac{1}{2}\partial_\alpha f_{\mu\nu}\partial^\alpha f^{\mu\nu} - \frac{1}{2}m^2 f_{\mu\nu}f^{\mu\nu} - \frac{1}{2}\partial_\alpha f^\mu_\mu \partial^\alpha f^\nu_\nu + \frac{1}{2}m^2 f^\mu_\mu f^\nu_\nu$$
$$- \partial^\alpha f_{\alpha\mu}\partial_\beta f^{\beta\mu} - f^\alpha_\alpha \partial^\mu\partial^\nu f_{\mu\nu} + f_{\mu\nu}J^{\mu\nu}_f .$$

$\Rightarrow$ D.O.F reduces from 10 to 5 by on-shell conditions

- Traceless: $f^\mu_\mu = 0$
- Transversal: $\partial_\mu f^{\mu\nu} = 0$

- How to deal with off-shell tensor in realistic processes?

Tensor: Stückelberg

- Fierz-Pauli conditions are not satisfied off-shell
- Stückelberg formalism: Introducing new fields and gauge symmetries
- Gauge-fixing decouples fields (free-field ϕ)
- Implemented into `WHIZARD` to analyze single D.O.F.

Assoziation of Stückelberg fields

- $f^{\mu\nu}$: On-shell $f^{\mu\nu}$
- ϕ: $\partial_\mu \partial_\nu f^{\mu\nu}$
- A^μ: $\partial_\nu f^{\mu\nu}$
- σ: $f^\mu{}_\mu$

$\rightarrow$ Feynman-gauge like propagators

Final Lagrangian

$$\begin{aligned}
\mathcal{L} =& \frac{1}{2} f_{f\mu\nu} \left(-\partial^2 - m_f^2\right) f_f^{\mu\nu} + \frac{1}{2} f_{f\;\mu}^{\mu} \left(-\frac{1}{2}\left(-\partial^2 - m_f^2\right)\right) f_{f\;\nu}^{\nu} \\
&+ \frac{1}{2} A_{f\mu} \left(\partial^2 + m_f^2\right) A_f^\mu + \frac{1}{2} \sigma_f \left(-\partial^2 - m_f^2\right) \sigma_f \\
&+ \left(f_{\mu\nu} - \frac{1}{\sqrt{6}} \sigma_f g_{\mu\nu}\right) J_f^{\mu\nu} \\
&- \left(\frac{1}{\sqrt{2} m_f} \left(A_{f\mu}\partial_\nu + A_{f\nu}\partial_\mu\right) - \frac{\sqrt{2}}{\sqrt{3} m_f^2} \sigma_f \partial_\mu \partial_\nu\right) J_f^{\mu\nu}
\end{aligned}$$

Resonance width and corresponding AQGC

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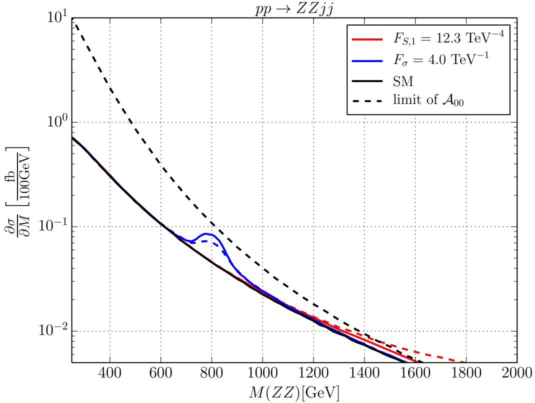
- Use width as parameter instead coupling ($\times m^3/(32\pi)F^2$)

	σ	ϕ	f	X
Γ	1	$1/4$	$1/30$	$1/120$

- Corresponding AQGC ($\times 32\pi\Gamma/m^5$)
  (transversal spin-2 coupling supressed)

	σ	ϕ	f	X
$F_{S,0}$	–	2	15	5
$F_{S,1}$	$\frac{1}{2}$	$-\frac{1}{2}$	-5	-35

Comparison of Simplified Models and EFT



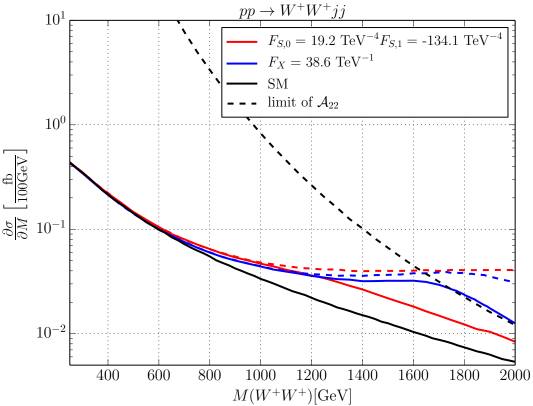
Calculation: `WHIZARD`

$$m_\sigma = 800\ \text{GeV}, \quad \Gamma_\sigma / m_\sigma = 0.1$$

$$F_{S,1} = \frac{1}{2}\frac{F_\sigma^2}{m_\sigma^2}$$

- EFT fails at resonance
- AQGC describes the rise of a resonance
- Energy validity range of theory is increased

Comparison of Simplified Models and EFT



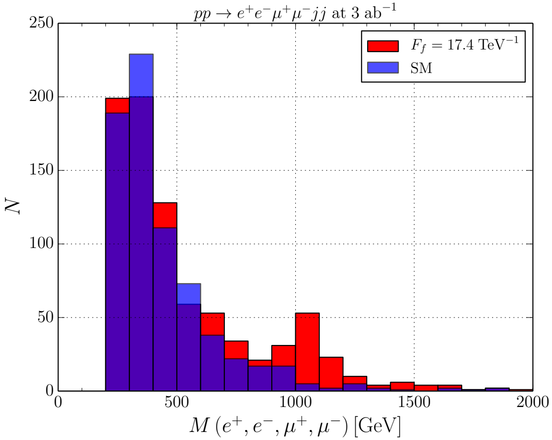
- EFT fails at resonance
- AQGC describes the rise of a resonance
- Energy validity range of theory is increased

Calculation: `WHIZARD`

$$m_X = 1800\ \text{GeV},\ \Gamma_X/m_X = 0.4$$

$$F_{S,0} = \frac{1}{2}\frac{F_f^2}{m_f^2},\ F_{S,1} = -\frac{1}{6}\frac{F_f^2}{m_f^2}$$

Complete LHC process at $\sqrt{s} = 14$ TeV



Simulation: `WHIZARD`

$m_f = 1.0$ TeV, $\Gamma_f / m_f = 0.1$

$F_{S,0} = \frac{1}{2}\frac{F_f^2}{m_f^2}, F_{S,1} = -\frac{1}{6}\frac{F_f^2}{m_f^2}$

Summary

- Effective theory: limited applicability for quartic gauge couplings
- Scheme to restore unitarity: Form-Factor or T-Matrix
- Frameworks for quantitative tests of the SM version of electroweak interactions which matches the low-energy EFT
- Realization: generic resonances $\rightarrow$ simplified model

$\Rightarrow$ Extension for EFT by resonances

	isoscalar	isotensor
scalar	✓	✓
tensor	✓	✓

Outlook

- Implementing T-matrix for transversal $\mathcal{O}_T$ and mixed $\mathcal{O}_M$ operators for VBS
  - $\mathcal{O}_{T,0}$, $\mathcal{O}_{T,1}$ and $\mathcal{O}_{T,2}$ in validation
- Include also $SU(2)_C$ violating operators
- Add transversal couplings for resonances
- Extend T-matrix for $1 \to 3$

Backup Slides

Overview of WHIZARD-models for VBS

- Model including all dim 6 operators of Warsaw basis: SM_dim6
- Models with T-matrix for longitudinal (transversal) couplings:

Model	SM-Higgs	Resonances	EFT representation
NoH_rx	X	Form factor	Non-linear
SM_rx	✓	Form factor	Non-linear
AltH	X	Fields	Non-linear
SSC	✓	Fields	Non-linear
SSC_2	✓	Fields	Linear
SSC_AltT	✓	Fields	Linear

! Resonances described by Form factos will neglect the induced transversal couplings of spin 2 particles (scalars are ok)

- The linear EFT-representation will give rise to couplings between Higgs and resonances or anomalous Higgs-VB and 4-Higgs couplings
- Model to calculate Isospin-Spin bounds: SM_ul

SINDARIN-options for SSC_2

Parameters	Default value	Description
fs0	0	Coupling strength of dimension eight operator $\mathcal{L}_{S,0}$ in TeV^{-4}
fs1	0	Coupling strength of dimension eight operator $\mathcal{L}_{S,1}$ in TeV^{-4}
gkm_s	0	Coupling strength of isoscalar scalar resonance in TeV^{-1}
gkm_p	0	Coupling strength of isotensor scalar resonance in TeV^{-1}
gkm_f	0	Coupling strength of isoscalar tensor resonance in TeV^{-1}
gkm_t	0	Coupling strength of isotensor tensor resonance in TeV^{-1}
cf	2	Arbitrary coupling parameter for isoscalar tensor
mkm_s	10^{10}	mass of isoscalar scalar resonance in GeV
mkm_p	10^{10}	mass of isotensor scalar resonance in GeV
mkm_f	10^{10}	mass of isoscalar tensor resonance in GeV
mkm_t	10^{10}	mass of isoscalar tensor resonance in GeV
wkm_s	0	width of isoscalar scalar resonance in GeV
wkm_p	0	width of isotensor scalar resonance in GeV
wkm_f	0	width of isoscalar tensor resonance in GeV
wkm_t	0	width of isoscalar tensor resonance in GeV
fkm	1	Flag to enable T-matrix unitarization (0:off, 1:on)
wres	1	Flag to set the width to Breit-Wigner width (0:off, 1:on, only if corresponding width = 0)

SINDARIN-options for SSC_AltT and SM_ul

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SSC_AltT

Parameters	Default value	Description
alt_tt	1	Flag for tensor-tensor (0:off, 1:on)
alt_tv	1	Flag for tensor-vector (0:off, 1:on)
alt_ts	1	Flag for tensor-scalar (trace) (0:off, 1:on)
alt_ts2	1	Flag for tensor-scalar (derivatives) (0:off, 1:on)

SM_ul

Parameters	Default value	Description
isa_00	0	Saturation of $\mathcal{A}_{00}$ (0:off, 1:on)
isa_02	0	Saturation of $\mathcal{A}_{02}$ (0:off, 1:on)
isa_11	0	Saturation of $\mathcal{A}_{11}$ (0:off, 1:on)
isa_20	0	Saturation of $\mathcal{A}_{20}$ (0:off, 1:on)
isa_22	0	Saturation of $\mathcal{A}_{22}$ (0:off, 1:on)
fkm	1	Flag to enable isospin spin saturation; 0:off, 1:on

Custodial Symmetry

$$\beta' \frac{v^2}{8} \text{tr}\left[T\mathbf{V}_\mu\right] \text{tr}\left[T\mathbf{V}^\mu\right]$$

- Free parameter $\beta' = \beta'(\rho_*)$
- Experimental data constrains $\rho_* = \frac{m_W^2}{c_w^2 m_Z^2}$:

$\rightarrow$ $\beta'(\rho_* \equiv 1) = 0$

- Impose approximate symmetry to forbid above term

$\Rightarrow$ $SU(2)_L \times U(1)_Y \rightarrow SU(2)_L \times SU(2)_R$

Fermionic sector

Very strong violation due large top mass

Bosonic sector

- Broken by coupling $B\tau_3 U \propto s_w^2$

$\Rightarrow$ Only small violation of $M_W = M_Z$

- Higgs mechanism: $SU(2)_L \times SU(2)_R \rightarrow SU(2)_C$

Lagrangian of Resonances

$$\mathcal{L}_\sigma = -\frac{1}{2}\sigma\left(M_\sigma^2 + \partial^2\right)\sigma + \sigma J_\sigma$$

$$\mathcal{L}_\phi = \frac{1}{2}\left[-\frac{1}{2}\,\mathrm{tr}\left[\Phi\left(m_\phi + \partial^2\right)\Phi\right] + \mathrm{tr}\left[\Phi J_\phi\right]\right]$$

$$\mathcal{L}_f = \mathcal{L}_{kin} - \frac{m_f^2}{2} f_{\mu\nu} f^{\mu\nu} + f_{\mu\nu} J_f^{\mu\nu}$$

$$\mathcal{L}_x = \mathcal{L}_{kin} - \frac{m_X^2}{4}\,\mathrm{tr}\left[\mathbf{X}_{\mu\nu}\mathbf{X}^{\mu\nu}\right] + \frac{1}{2}\,\mathrm{tr}\left[\mathbf{X}_{\mu\nu}\mathbf{J}_X^{\mu\nu}\right]$$

$$J_\sigma = F_\sigma^{||}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \mathbf{D}^\mu\mathbf{H}\right]$$

$$J_\phi = F_\phi^{||}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\mu\mathbf{H} - \frac{\tau^{aa}}{6}\,\mathrm{tr}\left[\left(\mathbf{D}_\mu\mathbf{H}\right)^\dagger \mathbf{D}^\mu\mathbf{H}\right]\right]$$

$$J_f^{\mu\nu} = F_f^{||}\left(\mathrm{tr}\left[\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \mathbf{D}^\nu\mathbf{H}\right] - \frac{c_f^{||}}{4} g^{\mu\nu}\,\mathrm{tr}\left[\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \mathbf{D}^\rho\mathbf{H}\right]\right)$$

$$J_X^{\mu\nu} = F_X^{||}\left[\frac{1}{2}\left(\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\nu\mathbf{H} + \left(\mathbf{D}^\nu\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\mu\mathbf{H}\right) - \frac{c_X^{||}}{4} g^{\mu\nu}\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \otimes \mathbf{D}^\rho\mathbf{H}\right.$$
$$\left. - \frac{\tau^{aa}}{6}\left(\mathrm{tr}\left[\left(\mathbf{D}^\mu\mathbf{H}\right)^\dagger \mathbf{D}^\nu\mathbf{H}\right] - \frac{c_X^{||}}{4} g^{\mu\nu}\,\mathrm{tr}\left[\left(\mathbf{D}_\rho\mathbf{H}\right)^\dagger \mathbf{D}^\rho\mathbf{H}\right]\right)\right]$$

Isospin-Spin Eigenamplitudes

$$
\begin{aligned}
a(w^+w^+ \to w^+w^+) &= a_{02}(s) - 10a_{22}(s) \\
&\quad + 15a_{22}(s)\frac{t^2+u^2}{s^2} \\
a(w^+w^- \to zz) &= \frac{1}{3}\left(a_{00}(s) - a_{20}(s)\right) - \frac{10}{3}\left(a_{02}(s) - a_{22}(s)\right) \\
&\quad + 5\left(a_{02}(s) - a_{22}(s)\right)\frac{t^2+u^2}{s^2} \\
a(w^+z \to w^+z) &= \frac{1}{2}a_{20}(s) - 5a_{22}(s) \\
&\quad + \left(-\frac{3}{2}a_{11}(s) + \frac{15}{2}a_{22}(s)\right)\frac{t^2}{s^2} \\
&\quad + \left(\frac{3}{2}a_{11}(s) + \frac{15}{2}a_{22}(s)\right)\frac{u^2}{s^2}
\end{aligned}
$$

Isospin-Spin Eigenamplitudes

$$
\begin{aligned}
a(w^+w^- \to w^+w^-) &= \frac{1}{6}\left(2a_{00}(s) + a_{20}(s)\right) - \frac{5}{3}\left(2a_{02}(s) + a_{22}(s)\right) \\
&+ \left(5a_{02}(s) - \frac{3}{2}a_{11}(s) + \frac{5}{2}a_{22}(s)\right)\frac{t^2}{s^2} \\
&+ \left(5a_{02}(s) + \frac{3}{2}a_{11}(s) + \frac{5}{2}a_{22}(s)\right)\frac{u^2}{s^2} \\
a(zz \to zz) &= \frac{1}{3}\left(a_{00}(s) + 2a_{20}(s)\right) - \frac{10}{3}\left(a_{02}(s) + 2a_{22}(s)\right) \\
&+ 5\left(a_{02}(s) + 2a_{22}(s)\right)\frac{t^2 + u^2}{s^2}
\end{aligned}
$$

Bounds on Eigenamplitudes

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AQGC amplitudes (GBET):

$$a_{00}(s) = \frac{1}{6}\left(7F_{S,0} + 11F_{S,1}\right)s^2$$

$$a_{02}(s) = \frac{1}{30}\left(2F_{S,0} + F_{S,1}\right)s^2$$

$$a_{11}(s) = \frac{1}{12}\left(F_{S,0} - 2F_{S,1}\right)s^2$$

$$a_{20}(s) = \frac{1}{3}\left(2F_{S,0} + F_{S,1}\right)s^2$$

$$a_{22}(s) = \frac{1}{60}\left(2F_{S,0} + F_{S,1}\right)s^2$$

a_{20} bounds

$$F_{S,0} = F_{S,1} = 480\ \text{TeV}^{-4} = (0.214\ \text{TeV})^{-4}$$

$$\sqrt{s} \lesssim 2.95 \cdot F_{S,0}^{-\frac{1}{4}} \approx 0.65\ \text{TeV}$$

$$\sqrt{s} \lesssim 3.50 \cdot F_{S,1}^{-\frac{1}{4}} \approx 0.75\ \text{TeV}$$

- Bounds depend on linear combination of AQGC
  (Assumption: Isospin/$SU(2)_C$ is preserved)

Algorithm for T-matrix

- Start with input model
- $\mathcal{L}_{S,1} = F_{S,1} \ \mathrm{tr}\left[\left(\mathbf{D}_\mu \mathbf{H}\right)^\dagger \mathbf{D}^\mu \mathbf{H}\right] \cdot \mathrm{tr}\left[\left(\mathbf{D}_\nu \mathbf{H}\right)^\dagger \mathbf{D}^\nu \mathbf{H}\right]$
- leads to the Feynman rules in unitary gauge

$$W^+_{\mu_1} W^+_{\mu_2} W^-_{\mu_3} W^-_{\mu_4} : \qquad \frac{\mathrm{i} g^4 v^4}{8} \left[F_{S,1} \left(g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3}\right)\right]$$

- Extract strong-interaction part in Goldstone limit (Feynman Rules)

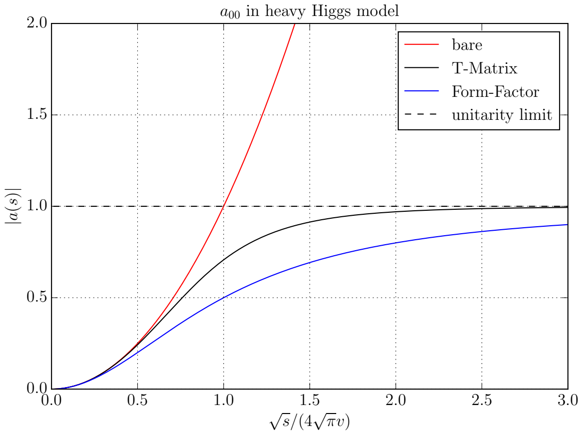
$$z(p_1) z(p_2) w^+(p_3) w^-(p_4) : \qquad 2\mathrm{i} F_{S,1} \left(p_1 \cdot p_2\right) \left(p_3 \cdot p_4\right)$$

- Use of custodial/crossing symmetry to calculate $a^0_{I\ell}$
- Unitarize via T Matrix projection: $a_{I\ell}(s) = \left[\mathrm{Re}\left(a^0_{I\ell}(s)^{-1}\right) - \mathrm{i}\right]^{-1}$
- Calculate counter terms: $\Delta a_{I\ell} = a_{I\ell} - a^0_{I\ell}$
- Re-insert s-channel correction as form factor into Feynman rules

\+ Extrapolate off-shell

$$W^\pm_{\mu_1} W^\pm_{\mu_2} \to W^\pm_{\mu_3} W^\pm_{\mu_4} : \quad 8\pi g^4 v^4 \left[\left(\Delta a_{02}(s) - 10\Delta a_{22}(s)\right) \frac{g_{\mu_1\mu_2} g_{\mu_3\mu_4}}{s^2} + 15\Delta a_{22}(s) \frac{g_{\mu_1\mu_3} g_{\mu_2\mu_4} + g_{\mu_1\mu_4} g_{\mu_2\mu_3}}{s^2}\right]$$

Comparison of Higgsless amplitude



! To calculate Λ_{FF}, the limit $|a_\ell| < 1$ was used instead of the conventional $\mathrm{Re}\,(a_\ell) < 0.5$

Comparison for $pp \to e^+\nu_e\mu^+\nu_\mu jj$ (p_T)

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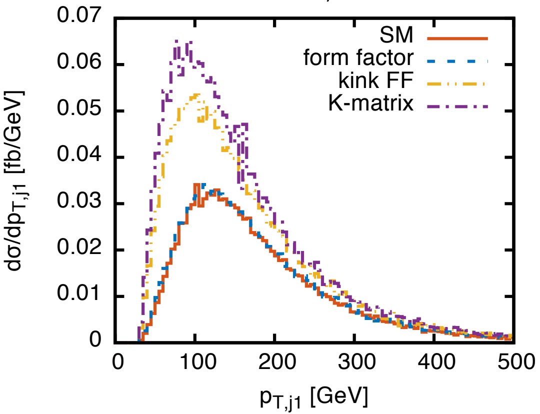

provided by M.Rauch (VBFNLO)

- EFT parameters: $F_{S,1} = 400\,TeV^{-4}$
- FF parameters: $p = 2,\ \Lambda_{FF} = 832\ \mathrm{GeV}$
- Kink: $\Theta\left(\Lambda_{FF} - M_{4l}\right) + \frac{\Lambda_{FF}^4}{M_{4l}^4}\Theta\left(M_{4l} - \Lambda_{FF}\right)$

Conversions

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Non-linear representation

Applequist, Bernard 1980

$$\mathcal{L}_{\alpha_4} = \alpha_4 \, \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}_\nu\right] \mathrm{tr}\left[\mathbf{V}^\mu \mathbf{V}^\nu\right]$$
$$\mathcal{L}_{\alpha_5} = \alpha_5 \, \mathrm{tr}\left[\mathbf{V}_\mu \mathbf{V}^\mu\right] \mathrm{tr}\left[\mathbf{V}_\nu \mathbf{V}^\nu\right]$$

Higgs-Doublet representation

Rauch, Zeppenfeld

$$\mathcal{O}_{S,0} = \frac{f_{S,0}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}_\nu\Phi\right]\left[\left(\mathbf{D}^\mu\Phi\right)^\dagger \mathbf{D}^\nu\Phi\right]$$
$$\mathcal{O}_{S,1} = \frac{f_{S,1}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}^\mu\Phi\right]\left[\left(\mathbf{D}_\nu\Phi\right)^\dagger \mathbf{D}^\nu\Phi\right]$$
$$\mathcal{O}_{S,0} = \frac{f_{S,0}}{\Lambda^4}\left[\left(\mathbf{D}_\mu\Phi\right)^\dagger \mathbf{D}_\nu\Phi\right]\left[\left(\mathbf{D}^\nu\Phi\right)^\dagger \mathbf{D}^\mu\Phi\right]$$

Conversions

$$F_{S,0} = 16\frac{\alpha_4}{v^4} = \frac{f_{S,0} + f_{S,2}}{\Lambda^4}, \quad \text{with } f_{S,0} = f_{S,2}$$
$$F_{S,1} = 16\frac{\alpha_5}{v^4} = \frac{f_{S,1}}{\Lambda^4}$$

Keep in mind: S_0 (S_2) and S_1 contribute also to anomalous *VVHH* and *HHHH* couplings!