Perturbative QCD for collider physics: recent developments

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Outline

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  ◦ Higgs production with $b\bar{b}$: choose the right scale
  ◦ no sbottoms in $p\bar{p} \rightarrow B + X$
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  ◦ $Z, W$ production
  ◦ Higgs production at the LHC
  ◦ Higgs coupling extractions

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Introduction: challenges

- In about a year, LHC begins its first physics run offering unprecedented opportunities.
- Two distinct features: high luminosity and high energy.
- Enormous rates for SM processes; can be used to study SM; have to be understood since they are backgrounds to New Physics.
- Factorization theorem

\[ \sigma^O = D_k^O \otimes \sigma_{k,j} \otimes F_j. \]

  - \( F_j \) describes hadron-parton transition \( \rightarrow \) Data;
  - \( \sigma_{k,j} \) describes parton-parton transition \( \rightarrow \) pQCD;
  - \( D_k \) describes “fragmentation” \( \rightarrow \) models, data, etc.
- pQCD is central for hadron collider phenomenology.
Introduction: challenges

• extraction of parton distribution functions
  ○ reliability
  ○ precision
• shower event generators
  ○ harder showers
  ○ combining with fixed order computations
  ○ hadronization models
• resummations
  ○ analytic resummations; numeric resummations
• NLO computations
  ○ higher multiplicity processes
• NNLO computations
  ○ general algorithms for NNLO calculations
  ○ NNLO phenomenology
All orders/leading order

- \( pp \rightarrow N \text{ jets} + X, \; N \leq 10 \) is a typical background process at the LHC.
- To deal with these multi-jet processes, we use all-purpose shower event generators, e.g. PYTHIA, HERWIG. Are these descriptions accurate?
- Showers are based on \textit{collinear} emissions.
- Collinear emissions are independent \( \Rightarrow \) probabilistic description.
- Showers are good for processes dominated by soft/collinear radiation.
- Showers generate large transverse momenta by emissions of many jets with moderate \( p_\perp \Rightarrow \alpha_s \) suppression of high \( p_\perp \) radiation.
- Shower do not change normalizations of total cross-sections

\[
\int d\sigma_{LO} \times MC = \sigma_{LO}.
\]

- \textbf{An alternative:} exact matrix elements for \( ij \rightarrow N \text{ jets} \). How do these things compare?
All orders/leading order

\[ M_{\text{eff}} = \sum_{\text{jets}} p_\perp + E_{\text{miss}} \]

- ALPGEN: exact matrix elements; correct hard emissions built in.
- PYTHIA: emulates hard emissions by producing large number of softer jets.
- PYTHIA underestimates the background significantly.
Acceptances for $pp \rightarrow W^- \rightarrow e\bar{\nu}$.

$A_W = \frac{1}{\sigma_{\text{tot}}^{e,\text{min}}} \int_{p_{\perp,\text{min}}^e} dp_{\perp} \frac{d\sigma}{dp_{\perp}^e}$. 

Mangano, Frixione

- NLO is just LO ($pp \rightarrow W + \text{jet} \rightarrow e\bar{\nu} + \text{jet}$) for $p_{\perp,\text{min}}^e > m_W / 2$.

$$\frac{A_W^{[\text{NLO}]}}{A_W^{[\text{HERWIG}]}} \sim 2 - 10, \quad \text{for} \quad p_{\perp,\text{min}}^e > 50 \text{ GeV}.$$
All orders/leading order: CKKW

- An \( N + 1 \)-jet event is obtained from an \( N \)-jet event either by
  large angle hard emission or shower.
- Event generators can do a better job for multi-jet processes if both mechanisms are taken into account.
- Catani-Krauss-Kuhn-Webber (CKKW) procedure:
  - calculate \( pp \to m \) HARD jets, with \( m < N \). Determine probability of an event with \( m \) hard jets using the cross-section values,
    \[
    P_m = \frac{\sigma_m}{\sigma_0 + \sigma_1 + \sigma_2 + \ldots \sigma_N}, \quad \sigma_m = \sigma_m(y_{\text{cut}}).
    \]
  - Generate hard jet configuration according to the probability distribution; shower it.
  - Requires introduction of a measure to distinguish between hard jet and shower jet.
- This procedure is being currently implemented in major shower event generators, such as PYTHIA and HERWIG.
Leading order: uncertainties

- Any leading order prediction has the renormalization and factorization scales uncertainty.
- $pp \rightarrow \nu \bar{\nu} + N \text{ jets}; \ p^j_\perp > 80 \text{ GeV}; |\eta| < 2.5.$
- $\mu = \sqrt{M_z^2 + \sum_{\text{jets}} p^2_\perp}; \ \mu_r = \mu_f = \mu/2...2\mu.$

<table>
<thead>
<tr>
<th>N</th>
<th>$\sigma(2\mu) \text{ pb}$</th>
<th>$\sigma(\mu/2) \text{ pb}$</th>
<th>variation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>182</td>
<td>216</td>
<td>17%</td>
</tr>
<tr>
<td>2</td>
<td>47.1</td>
<td>75.4</td>
<td>46%</td>
</tr>
<tr>
<td>3</td>
<td>6.47</td>
<td>13.52</td>
<td>70%</td>
</tr>
<tr>
<td>4</td>
<td>0.90</td>
<td>2.48</td>
<td>93%</td>
</tr>
</tbody>
</table>

Next-to-leading order computations are necessary.
Next-to-leading order

- The NLO prediction is often the first quantitative prediction.
- Typical background \((t\bar{t})^n (WZ)^m\) jets\(^l\), \(n, m, l > 0\).
- Current state of the art is \(2 \rightarrow 3\) processes:
  - NLOJET++ [Nagy] \(pp \rightarrow (2, 3)j, \ ep \rightarrow 3j, \ e^+e^- \rightarrow 3, 4j, \gamma^*p \rightarrow (2, 3)j;\)
  - AYLEN/EMILIA [de Florian, Dixon, Kunszt, Signer] \(pp \rightarrow (W, Z) + (W, Z, \gamma);\)
  - MCFM [Campbell, Ellis] \(pp \rightarrow (W, Z) + (0, 1, 2)j, \ pp \rightarrow (W, Z) + b\bar{b};\)
  - DIPHOX/EPHOX [Aurinche et. al] \(pp \rightarrow \gamma + 1j, \ pp \rightarrow \gamma\gamma, \gamma^*p \rightarrow \gamma + 1j;\)
  - VBFNLO [Figy, Zeppenfeld, Oleari] \(pp \rightarrow (W, Z, H) + 2j;\)
- Flexible programs: arbitrary restrictions on the final state can be applied.
- We want to extend the NLO computations to \(2 \rightarrow 4, 5, \text{etc.}\) processes.
- Problem: one-loop \(5, 6, 7...n\)-point functions.
  - Direct numerical integration is not possible because those functions have soft and collinear divergences.
  - Simplifications of many-point functions produce fictitious singularities that are hard to handle.
Next-to-leading order

- **Recent progress (technical):**
  - Mellin-Barnes transform
  - IBP’s, sector decompozition, numerics
  - Numerical solutions of IBP’s
  - Bernstein-Tkachov theorem
  - Integration in momentum space

- **Recent progress (calculations):**
  - $pp \rightarrow H \rightarrow 2$ jets (virtual), Zanderighi, Giele, Ellis;
  - $pp \rightarrow t \rightarrow Wb$, Ellis, Campbell;
  - $pp \rightarrow H\bar{b}, H\bar{t}$, Dawson, Jackson, Wackeroth, Reina, Spira, Krämer;

- **First complete $2 \rightarrow 4$ computation:** $e^+e^- \rightarrow 4$ fermions, Denner, Dittmaier et al.

- **Flexible methods are needed; must be easily adaptable to New Physics models.**
Next-to-leading order

- Consider Higgs production in association with $b$ quarks. Two options:

  $$\text{PT in } \frac{\alpha_s}{\pi} \ln\left(\frac{m_H^2}{m_b^2}\right)$$

  $$\text{PT in } \frac{\alpha_s}{\pi}$$

- Puzzle: $\sigma_{LO}(gg \rightarrow b\bar{b}H) \sim 0.1 \sigma_{LO}(b\bar{b} \rightarrow H)$.

- Resolution: $\mu_F = m_H/4$ is an appropriate scale (kinematics).

  Willenbrock, Maltoni, Plehn, Boos

- This prediction is confirmed by explicit (later) higher order calculations.
Next-to-leading order

- $b\bar{b} \rightarrow H$ is currently known through NNLO; $\mu_F = m_H/4$ is the right scale!
  
  Harlander, Kilgore

- $gg \rightarrow b\bar{b}H$ is currently known through NLO; compares well with $b\bar{b} \rightarrow H$.
  
  Dawson, Jackson, Reina, Wackeroth, Krämer, Spira

- Gain confidence from looking at the same process in different ways.
NLO: bottom production

- Bottom production in hadron collisions: $p\bar{p} \rightarrow B + X$ was a long-standing problem for pQCD with discrepancy often quoted as a factor 2-4.
- New Physics explanations, e.g. light gluinos, sbottoms.

NLO QCD prediction for $p_B^\perp$ is non-trivial:
  - $b \rightarrow B$ fragmentation function;
  - large uncertainties due to PDFs;
  - large NLO QCD corrections;
  - $\sigma_{\text{tot}}$ is dominated by $p_\perp \sim m_b$.

Cacciari, Nason

- Excellent agreement of the total cross-sections

$$\sigma_{J/\psi}^{\text{CDF}} = 19.9^{+3.8}_{-3.2} \text{ nb}, \quad \sigma_{J/\psi}^{\text{PQCD}} = 19.0^{+8.4}_{-6.0} \text{ nb}.$$  

Cacciari et al.

- Large $\pm50\%$ theory uncertainty remains.
Event generators and higher orders

- Shower event generators and perturbative calculations are complimentary:
  - Showers: universal, realistic jets, automatic resummations, hadronization;
  - PT: correct rates, correct description of hard emissions, improvable errors.
- Combining MC’s and perturbative computations is a good (old) idea
- The most advanced implementation is called MC@NLO (based on HERWIG shower):

\[ MC@NLO = MC \left( 1 + \alpha_s [NLO - MC_{\alpha_s}] \right). \]

Features:
- outputs unweighted events;
- no double counting;
- total rates are accurate through NLO.

Processes included:
- $H$, $W$, $Z$, $VV$, $HZ$, $t\bar{t}$, $b\bar{b}$ and single top.

Alternative implementations would be most useful

Krämer, Nagy, Soper
NNLO

- NNLO calculations are desirable for:
  - processes where good estimate of the uncertainty is required;
  - processes with large NLO corrections.
- This leaves us with $H, W, Z, 2\text{ jets}, \text{ heavy quarks}$.
- What is known through NNLO for hadron colliders:
  - $W, Z, gg \rightarrow H, gg \rightarrow A, \bar{b}b \rightarrow H$ production; total cross-sections; 
    van Neerven, Matsuura, Kilgore, Harlander, Anastasiou, K.M., Ravindran, Smith
  - $W, Z, \gamma^*$ rapidity distribution; 
    Anastasiou, Dixon, K.M., Petriello
  - $gg \rightarrow H, Z, W$ production, fully differential with spin correlations; 
    Anastasiou, K.M., Petriello
- Generalization to $2 \rightarrow 2$ processes (jets, heavy quarks) is highly non-trivial.
NNLO: PDFs

- A consistent implementation of NNLO calculations requires NNLO PDFs and NNLO evolution kernels.
- NNLO Altarelli-Parisi splitting kernels known. Vermaseren, Moch, Vogt
- NNLO PDFs extractions exist. MRST, Alekhin.
- Broad measure of PDFs fits reliability:

\[
\alpha_s^{\text{Alekhin}}(M_Z) = 0.114(1), \quad \alpha_s(M_Z) = 0.121(1).
\]

NNLO effects increase the disagreement.
- For hard processes at the LHC, PDF uncertainty is

\[
\frac{\delta\sigma}{\sigma} \approx 5\%, \quad M \sim 100 \text{ GeV}, \quad |Y| < 2.
\]
- For larger $|Y|$, $\ln(1/x)$ terms may require resummations (BFKL, saturation)
NNLO: $Z$ and $W$ production

- Use the $Z, W$ production to measure $L$.

- Partonic luminosities $\leftrightarrow$ rapidity of gauge bosons

\[
\frac{d\sigma}{dM \, dY} \sim q_1(x_1)q_2(x_2), \quad x_{1,2} = \frac{M}{\sqrt{S}} e^{\pm Y}.
\]

- NNLO results: scale stability and PDF sensitivity

Dittmar et al.

Anastasiou, Dixon, Petriello, K.M.
NNLO: $W^-$ production

- The knowledge of rapidity distributions of $Z$, $W$ bosons is insufficient for deriving lepton distributions because of spin correlations.

- The fully differential NNLO QCD calculation for $pp \rightarrow e + \bar{\nu} + X$ is now available. Cuts of the form (ATLAS, CMS)

  \[
  \begin{align*}
  \text{Cut1} & \quad p_{\perp}^e > 20 \text{ GeV}, \quad |\eta_e| < 2.5, \quad E_{\text{miss}} > 20 \text{ GeV} \\
  \text{Cut2} & \quad p_{\perp}^e > 40 \text{ GeV}, \quad |\eta_e| < 2.5, \quad E_{\text{miss}} > 40 \text{ GeV}
  \end{align*}
  \]

<table>
<thead>
<tr>
<th>LHC</th>
<th>$A(\text{MC@NLO})$</th>
<th>$\frac{\sigma_{\text{MC@NLO}}}{\sigma_{\text{NLO}}}$</th>
<th>$A(\text{NNLO})$</th>
<th>$\frac{\sigma_{\text{NNLO}}}{\sigma_{\text{NLO}}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut1</td>
<td>0.485</td>
<td>1.02</td>
<td>0.492</td>
<td>0.983</td>
</tr>
<tr>
<td>Cut2</td>
<td>0.133</td>
<td>1.03</td>
<td>0.155</td>
<td>1.21</td>
</tr>
</tbody>
</table>

- 1 – 2 percent NNLO effects for $p_{\perp}^{e,\text{min}} > 20 – 30 \text{ GeV}$;
- 10 – 20 percent NNLO effects for $p_{\perp}^{e,\text{min}} > 40 – 50 \text{ GeV}$.  
  Petriello, K.M.

- For Cut2, MC@NLO gets the acceptance wrong since second hard emission is important.
NNLO: Higgs boson signal at the LHC

- QCD effects increase the inclusive $gg \to H$ production cross-section by a factor two.
- For $H \to \gamma\gamma$, the following cuts on the final photons are imposed (ATLAS,CMS):
  - $p_{T}^{(1)} \geq 25 \text{ GeV}$, $p_{T}^{(2)} \geq 40 \text{ GeV}$, $|\eta_{1,2}| \leq 2.5$.
  - Isolation cuts, e.g. $E_{T,\text{had}} \leq 15 \text{ GeV}$, $\delta R = \sqrt{\delta \eta^2 + \delta \phi^2} < 0.4$.

- Do the conclusions based on inclusive calculations change when those cuts are imposed?

C. Anastasiou, K.M., F. Petriello

Re-weighting MC@NLO and PYTHIA to double differential distribution in Higgs $p_{T}$ and rapidity. [Davatz et al.]
Higgs coupling extractions

- Analyses of Higgs coupling use relation
  \[ \sigma(H) \times \text{Br}(H \rightarrow X) = \frac{\sigma_{gg}^{\text{SM}}}{\Gamma_{gg}^{\text{SM}}} \times \frac{\Gamma_{gg}}{\Gamma_{\text{tot}}}. \]

- Calculate and assign theoretical uncertainty to \( \sigma_{gg}^{\text{SM}}/\Gamma_{gg}^{\text{SM}} \), extract \( \Gamma_{gg} \Gamma_{X}/\Gamma_{\text{tot}} \); new states in loops drop out.

- Studies assign ±20% uncertainty to \( \sigma/\Gamma \) for \( gg \rightarrow H \) production mode. Dührssen et al.

\[ \Gamma^{\text{SM}} = \alpha_s(\mu_r)^2 C_1(\mu_r)^2 [1 + \alpha_s(\mu_r)X_1 + ...]; \]
\[ \sigma^{\text{SM}} = \alpha_s(\mu_r)^2 C_1(\mu_r)^2 [1 + \alpha_s(\mu_r)Y_1 + ...]. \]

- Scale variation correlated; large \( \mu_r \) variations cancel; \( \Delta(\sigma/\Gamma) = \pm 5\% \).

- Recent developments:
  - N^3LO soft+virtual corrections to \( \sigma_{gg} \rightarrow H \) Moch, Vermaseren, Vogt
  - N^3LO corrections to \( \Gamma_{gg} \) Baikov, Chetyrkin
  - \( \Delta\sigma : \pm 10\% \rightarrow \pm 4\%; \quad \Delta\Gamma : \pm 5\% \rightarrow \pm 2\%. \)
Conclusions

- Good understanding of pQCD is an important pre-requisite for the successful LHC physics program.
- Recent developments include
  - making showers more realistic (harder);
  - large-scale NLO computations;
  - merging shower event generators and NLO computations;
  - emerging NNLO phenomenology.
- From existing computations and comparison with data we should learn
  - to appreciate uncertainties;
  - to understand when popular techniques are applicable;
  - to choose “right” scales in perturbative predictions;
  - to avoid rushy conclusions if something does not add up.
- There are plenty of challenges, room for new ideas and unorthodox approaches even in Old Physics. A significant progress that occurred in pQCD in the last few years will be very useful once the LHC turns on.